Advances in Global Positioning System Technology for Geodynamics Investigations: 1978-1992

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Over the last decade, users of the Global Positioning System (GPS) have developed the technology capable of meeting the stringent requirements to study geodynamics signals. Positioning precision is currently at the millimeter level on local scales (~10 km), and at the centimeter level on global scales (~10,000 km). Now that the GPS constellation is nearing completion and techniques are maturing, the doors are open to deeper investigations of geophysical phenomena covering a broad temporal and spatial spectrum. After a decade of great strides in improving geodetic precision, the focus is moving to operational systems and quality control. Several GPS analysis groups are now routinely estimating Earth polar motion parameters and geocentric coordinates of 20-30 stations on a daily basis with centimeter accuracy, besides providing precise estimates of GPS satellite positions to support regional geodynamics investigations. Within the next few years, this analysis will grow to include the computation of geocentric coordinates for ~200 stations, every day. Hence, GPS will allow us to view the Earth as a 200-vertex, deforming, rotating polyhedron.

INTRODUCTION

Over the last few years, various investigators have demonstrated centimeter-level precision using GPS not only over local scales (tens of km), but also over regional scales (hundreds of km) and, more recently, global scales (thousands of km). Each of these scales has its own regime of scientific applications and, correspondingly, its own set of technical developments that deliver the required accuracy.

Table 1 summarizes some of these applications, requirements, and a list of techniques appropriate to each scale. This is not an exhaustive list, but it simply helps to put things in perspective. Clearly, GPS has major contributions to offer science where other techniques may be more costly and inconvenient. However, achieving the highest precision for regional and global scales does come at a cost. As we shall see, we must take a degree of care, which is not normally required for local networks, in the design of regional networks, treatment of GPS orbits, parameter estimation strategy, and ambiguity resolution. Kinematic, pseudokinematic, and rapid static surveying represent more efficient means to conduct local surveys; these methods have required their own class of technical development.

The last decade has witnessed the rise of the Global Positioning System (GPS) as the primary tool of geodetic investigators. This has occurred for 4 major reasons: (1) easy

and economical access to GPS hardware and software; (2) portability of field equipment; (3) international collaboration, which is beneficial from both an economical and a global-geodetic point of view; and (4) dramatic improvements in techniques for high-precision GPS geodesy.

This paper summarizes some of the major technological advances that have brought GPS to the forefront of geodetic investigations. The precision of GPS geodetic baselines has improved by 3 orders of magnitude over the last decade. Figure 1 shows an exponential fit to GPS baseline precision taken from a sample of the literature over the last decade. The reason for this improvement can be explained by a combination of continuous refinement and breakthrough in analysis techniques, hardware, global network coverage, and the GPS satellite constellation (which is now almost complete). This paper documents these improvements, and as such, takes on a rather historical perspective.

It is doubtful that the curve in Figure 1 can be extrapolated like this, but it is clear that investigators that strive for the ultimate in precision are not short of ideas on ways to improve current precision. To give the reader some perspective on the current level of precision, and why this level of precision is important for geophysical studies, Figure 2 shows a co-seismic displacement signal of a few millimeters over a distance of 1700 km [Blewitt et al., 1993; Bock et al., 1993].

Scope of this Paper

In order to limit the length of this paper, it has been necessary to assume that the reader has already been introduced to GPS and its application to geodynamics [Bilham, 1991;

TABLE 1. GPS Applications, Precision, and Techniques as a Function of Network Scale

Scale	Geophysical Applications	Precision	Important Techniques
Very Local: $10^{-2} - 10^{0}$ km	3-dimensional geodetic ties for: • technique intercomparison • technique combination • reference frame unification.	0.1 – 1 mm (in 10^2 sec)	Broadcast orbits (assuming no S/A) Multipath calibration Kinematic surveying
Local: $10^{0} - 10^{2}$ km	Deformation in fault zones: • secular strain accumulation • seismic and aseismic slip • post-seismic relaxation • precursory signals ? Surface topographic change • volcanic uplift • glaciology and ice sheets • mountain/valley formation With airborne gravimetry, spatial and temporal variations of local gravity field	1 – 4 mm (in 10^3 sec)	Improved orbits Ionosphere-free phase Local troposphere models Regional network to resolve absolute troposphere delay Rapid static survey (with fast ambiguity resolution) Tracking of platforms (airborne & surface): • gravimetry • altimetry • interferometric SAR • photogrammetry
Regional: $10^{2} - 10^{3}$ km	Plate boundary structure Microplates (block rotation) Subduction and spreading zones Continental collision Intra-plate deformation Far-field seismic displacement Surface topographic change: • ice sheet volume change • mountain range formation	4 – 10 mm (in 10^4 sec)	Precise orbits (either supplied or estimated) Fiducial network to provide reference frame (and orbit estimation) Mix of baseline lengths (ambiguity resolution) Site-specific, stochastic, tropospheric delay Tracking of buoys for marine geodesy
Global: $10^{3} - 10^{4}$ km	Plate tectonics Excitation of Earth wobble and spin rate (atmosphere, oceans, etc.) Absolute geocentric height Sea-level change Post-glacial rebound Tidal effects Oceanic & atmospheric loading Sea-surface topography High resolution gravity field	~1 cm (in 10^5 sec)	Accurate satellite force modeling and orbit determination with global network (> 20 stations) Free-network solution Accurate Earth models Precise pseudorange for ambiguity resolution Daily estimation of polar motion and length-of-day Tracking of Earth orbiters (altimetry, gravimetry)

Dixon, 1991; Larson and Agnew, 1991, Larson et al., 1991; Davis et al., 1991; King and Blewitt, 1990; Blewitt et al., 1988; Blewitt, 1990c; Hager et al., 1991; Tsuji and Murata, 1992; Shimada and Bock, 1992]. This paper emphasizes ground-based geodesy; certain specialized topics, such as using GPS for tracking instrument platforms (e.g., airborne gravimetry), are beyond the scope of this review. Unfortunately, there is not sufficient space here to discuss the various geophysical experiments and scientific results. The reader may find these topics discussed in other papers within this monograph.

RECEIVER DEVELOPMENT

First Generation Receivers

In the late 1970's, several people realized that we could use the GPS satellites for precise relative positioning, similar to how we use quasars for very long baseline interferometry (VLBI). MacDoran [1979] proposed and developed a system

known as SERIES, which measured the group delay of GPS signals with the aid of a small antenna dish [Buennagel et al., 1984]. This approach proved to be cumbersome and inadequate for high-precision applications. Counselman and Shapiro [1979] proposed tracking several satellites simultaneously, emphasizing the importance of measuring the phase delay of the carrier signals. They proposed a geodetic system that contains the essential principles in use today [Bossler et al., 1980]. An omni-directional antenna could be used instead of an antenna dish, because carrier phase measurements are less susceptible to multipath (caused by signal reflections from local objects). Differencing the data between simultaneously observed satellites effectively eliminated clock bias, thereby dramatically reducing the requirements on the stability of the local oscillator.

Counselman et al. [1979] developed the omni-directional, dual-frequency MITES system, with which they demonstrated short baseline estimation [Counselman et al., 1981]. They demonstrated that carrier phase multipathing was not a serious
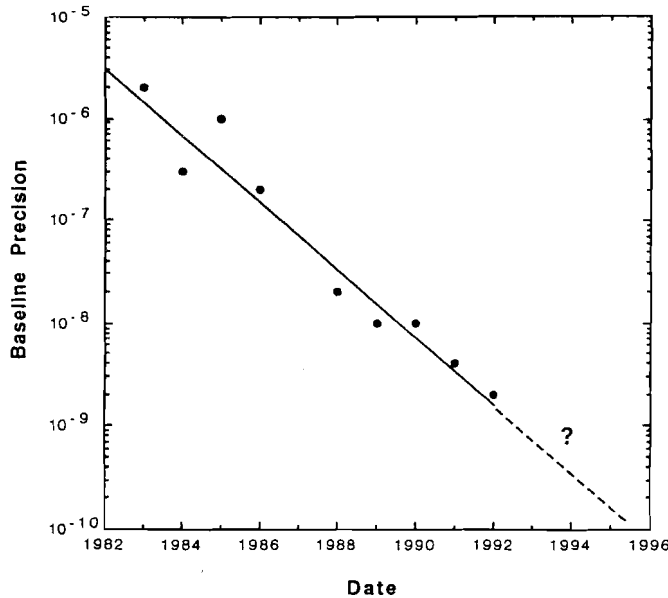
Fig. 1. Fit to GPS baseline precision as reported in the literature.

problem. The Macrometer system [Counselman, 1982] demonstrated baseline precision at the few-mm level over a few kilometers [Counselman et al. 1983; Goad and Remondi, 1984; Bock et al., 1984]. Also in the early 1980's, the U.S. Air Force (AFGL) developed dual-frequency, omni-directional, codeless receivers (based on the Macrometer), with additional support by the United States Geological Survey (USGS) and the National Aeronautical and Space Administration (NASA). King et al. [1984] used AFGL receivers in 1983 for a long baseline study (Richmond, FL to Haystack, MA). The Jet Propulsion Laboratory (JPL) began the SERIES-X project to develop an omni-directional receiver that provided dual-frequency phase and group delay measurements [Crow et al., 1984]. JPL used
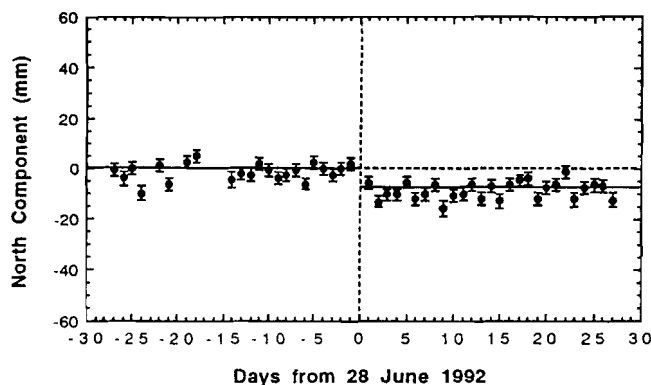

Fig. 2. Co-seismic displacement due to the Landers earthquake detected at the few-mm level over a distance of 1700 km between JPLM (California) and DRAO (British Columbia), [Blewitt et al., 1993; Bock et al., 1993].

the SERIES-X receiver several times during the mid-1980's to measure a 245-km baseline in California.

Despite their greater cost, dual-frequency systems became increasingly desirable for precise work, because they allowed for the removal of ionospheric delay bias from geodetic solutions (which is of the order 10^{-6} of baseline length) [Spilker, 1980]. Under contract with the Defense Mapping Agency, the National Oceanic and Atmospheric Administration, and USGS, Texas Instruments developed the first commercial receiver to produce dual-frequency pseudorange and carrier phase observables that were derived using knowledge of the P-code [Ward, 1980 and 1982; Henson et al., 1985]. A survey of the literature shows that the TI-4100 receiver was an important workhorse for precise GPS geodesy in the late-1980's [Beutler et al., 1987; Tralli et al., 1988; Prescott et al., 1989; Davis et al., 1989; Dong and Bock, 1989; Blewitt, 1989; Schutz et al., 1990; Freymueller and Kellogg, 1990; Murray et al., 1990; Larson and Agnew, 1991; Kornreich Wolf et al., 1990; Dixon et al., 1991].

On the practical side, receivers like the TI-4100 gave geodetic investigators easy access to the broadcast ephemerides, and receiver synchronization was possible in post-processing software using the pseudorange data. One of the principal reasons for developing the TI-4100 was that geodesists desired full-wavelength ambiguity on the L2 channel, which improves ambiguity resolution for long baselines [Bender and Larden, 1985]. The TI-4100 provided important data sets that researchers used to test new algorithms for data editing and long-baseline ambiguity resolution. For example, Figure 3 shows the TurboEdit algorithm [Blewitt, 1990a], which uses the pseudorange to automatically detect and correct cycle-slips in the widelane phase ($\phi 1-\phi 2$). However, the TI-4100 could track no more than 4 satellites simultaneously; with the launch of more GPS satellites, it became sub optimal for precise positioning applications.

Second Generation Receivers

The late 1980's saw the development of receivers that could simultaneously track 8 satellites or more. Codeless receivers by Trimble Navigation, Ashtech, and Aero Service (the Mini Mac) were coming into use. These receivers were highly
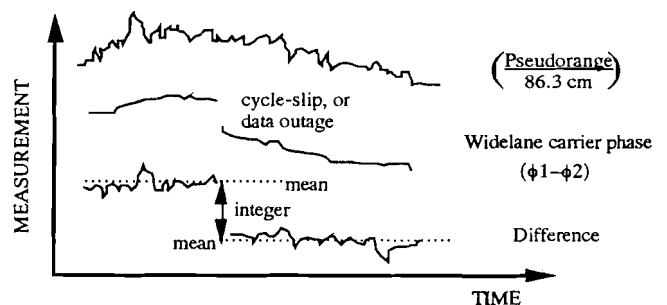

Fig. 3. A simplified illustration of the TurboEdit algorithm for detecting and correcting cycle-slips. An appropriate linear combination of P1 and P2 pseudorange is formed which theoretically matches the geometrical, ionospheric, and clock variations in phase delay [Blewitt, 1990a].

portable and easier to use in the field. Unlike first generation codeless receivers, they produced the more precise C/A carrier phase observable on the L1 channel. Today, such codeless receivers are more widely used than P-code receivers, although the trend is again reversing. The reason for this vacillation is probably due to industry's reluctance in the 1980's to develop receivers when the availability of P-code to the civilian community was (and is still) under question. The issue of whether the military will turn on P-code encryption, or Anti-Spoofing (A/S), has for many years cast a cloud of uncertainty over the future of P-code receivers. This uncertainty has spurred recent advances in extracting high-precision C/A code and P1–P2 observables under A/S conditions [Srinivasan et al., 1989]. Such technological advances may effectively eliminate the negative impact that Anti-Spoofing would otherwise have had on the precision and operational efficiency of GPS-based geodesy for geodynamics research.

The Rogue receiver was developed by JPL for NASA geodynamics investigations and ionospheric calibrations [Thomas, 1988]. Commercial derivatives (SNR-8, SNR-800) were produced by Allen Osborne and Associates. These high-precision P-code receivers were used with great success in the GIG'91 global geodetic campaign, demonstrating 12 mm RMS agreement with 33 VLBI station coordinates in the northern hemisphere [Blewitt et al., 1992]. Rogue-type receivers currently form the core network for the International GPS and Geodynamics Service. Although very precise and successful for permanent operations, the Rogue was never intended to be fieldworthy.

Partly due to the success of P-code receivers in achieving the most demanding requirements of geodynamics investigations, and due to new data processing algorithms that enhanced the efficiency of geodetic investigations (e.g., rapid static surveying), the early 1990's saw the commercial production of second-generation, highly portable, high precision P-code receivers. By developing the TurboRogue, NASA played a key role in spurring this renewed interest in P-code receivers. Like the Rogue, the TurboRogue (SNR-8000) produces dual-frequency pseudorange and full-wavelength L2 carrier phase even in the presence of A/S. Under A/S conditions, the TurboRogue receiver shows better precision for dual-frequency pseudorange than for the original TI-4100 P-code data, which was adequate for the baseline-independent resolution of widelane ambiguities and for the correction of integer-cycle discontinuities. These enhanced features may have been a factor in industry's reassessment of the risk in developing the new P-code receivers. Currently, several manufacturers are offering (or will soon be offering) second-generation P-code receivers.

PRECISE ORBIT DETERMINATION

The Fiducial Concept

Precise GPS geodesy requires an accurate knowledge of satellite positions. We can solve for GPS satellite orbits using GPS data recorded at sites with known coordinates (for example, at VLBI sites). Of course, orbit determination and the concept of using tracking stations at known locations are old ideas, but putting these ideas into practice established GPS as a serious tool for regional geodynamics investigations.

Pioneering demonstrations of GPS orbit determination for precise baseline estimation are described by King et al. [1984], Abbot et al. [1985], and Beutler et al. [1985], with reported precision of a few $\times\ 10^{-7}L$, where L is baseline length. In an experiment known as the "March 1985 High Precision Baseline Test," fiducial receivers were collocated at several VLBI sites across the United States, allowing analysts to constrain a subset of receiver locations at VLBI-inferred coordinates, simultaneously solving for GPS orbits and remaining receiver locations [Melbourne et al., 1985; Davidson et al., 1985; Bock et al., 1985b]. Results showed few-centimeter baseline precision over hundreds of kilometers [Bock et al., 1986a; Bock et al., 1986b; Lichten and Border, 1987].

Thornton et al. [1986] systematically studied the "fiducial concept" in developing a plan for crustal motion investigations in the Caribbean region. They derived error estimates for regional baseline determination using a variety of assumed fiducial geometry. History has shown remarkable agreement between their initial error estimates and the precision demonstrated in the Caribbean by Dixon et al. [1991]. Other detailed studies of the fiducial concept and its application to the study of crustal dynamics are presented by Dixon et al. [1985], Schutz et al. [1990], Larson et al. [1991], Murray et al. [1990], and Freymueller and Kellogg [1990].

Some recent studies have investigated systematic errors introduced by imposing fiducial constraints [Lichten et al., 1989, Larson et al., 1991]. They showed that, by varying the choice of fiducial network, regional solutions change at the level of $10^{-8}L$. As we shall discuss later, we can overcome such systematic errors by using a free-network approach [Beutler et al., 1986; Herring et al., 1991; Heflin et al., 1992].

Orbit Modeling and Estimation Strategy

Several investigators have described a variety of orbit estimation techniques [Beutler et al., 1987; Colombo, 1986, 1989; Schutz et al., 1990; Lindqwister et al., 1990]. Papers by Lichten and Bertiger [1989] and Lichten [1990a] discuss the importance of estimating stochastic non-gravitational forces on the GPS satellites (using a Kalman-type filter) if the estimated orbit arc is longer than 24 hours. In hindsight, the estimation of multi-day arcs in the late 1980's was an interim solution to improving orbit accuracy at a time when continental-scale fiducial networks were being used, the global network was sparse, and there were few GPS satellites. With the advent of a true global distribution of tracking stations in 1992, 24-hour orbit arcs are sufficient to produce baseline precision of $10^{-9}L$ [Heflin et al., 1992].

Since the GPS satellite solar panels have a large surface area to mass ratio, solar radiation pressure produces significant accelerations. Most precise geodetic software today use models described by Fliegel et al. [1985], Fliegel and Gallini

[1989] and Fliegel et al. [1991]. It remains an outstanding problem in GPS orbit determination to model eclipsing satellite trajectories adequately [Lichten, 1990a; Schutz et al., 1990]. Vigue and Schutz [1990] and Feltens and Groten [1990] have conducted preliminary studies into thermal effects, which may be the key to this problem.

Routine, High-Precision Orbit Production

Starting June 1992, several analysis groups participating in the International GPS and Geodynamics Service (IGS) have been (electronically) publishing precise GPS orbit positions every week, which regional geodynamics investigators can then use to achieve baseline accuracy of $10^{-8}L$. The transfer of this burden to a few dedicated groups has numerous advantages: (1) on any given day, there are solutions by several independent groups which can be intercompared to assess accuracy; (2) satellite and station problems are more readily identified, and the user will be warned of anomalies; (3) the satellite positions are all given in the International Terrestrial Reference Frame (ITRF), ensuring a level of compatibility between the different regional analyses; and (4) the alternative would require that every regional investigator go through the time-consuming process of estimating and assessing the accuracy of GPS orbits for each experiment.

TROPOSPHERIC MODELING AND ESTIMATION

Tropospheric delay has a wet and dry component [Davis et al., 1985]. The dry component is ~2 m at zenith, and varies slowly and smoothly (since it is largely determined by air pressure). Atmospheric water vapor causes the wet component, which is only ~10 cm at zenith, but it has random variations over a broad spatial and temporal scale (since the variations are related to turbulence). It is important to model these delays accurately, especially for the estimation of vertical station coordinates. We can theoretically map wet and dry delays to an equivalent zenith delay using a mapping function (one for wet, one for dry) [Lanyi, 1984, Davis et al., 1985]. We can then estimate a residual zenith delay bias using the GPS data themselves. Alternatively, we can constrain the zenith delay to values determined by surface meteorological instruments or water vapor radiometers (WVR's) [Ware et al., 1986; Rocken et al., 1991; Johansson, 1992].

Stochastic Estimation

Analyses of various experiments have demonstrated that stochastic estimation of a zenith tropospheric bias (using a Kalman-type filter) produces comparable results to using a WVR in a variety of regions (including California, Mexico, Caribbean, Central and South America). In some circumstances (e.g., rainy conditions), WVR's are ineffective; thus, stochastic estimation tends to be a more robust approach from this point of view. Moreover, WVR's are currently bulky, heavy, and are very expensive (>$100K) if we require a retrieved delay accuracy of 5 mm.

An approach that appears to work well both in VLBI and GPS is to estimate a zenith bias as a random walk process, which Figure 4 illustrates. In this figure, the dots represent estimates of the zenith tropospheric bias, and are derived using a batch sequential filter as follows. Let us take the black dot on the left as the current estimate. We can map this estimate forward in time to provide a future estimate, represented by the white dot. We then add (in quadrature) the error bar to random-walk process noise, which grows as the square root of time. The lined dots represent a future, independent data estimate. A weighted average of this data estimate and the mapped estimate provides a new current estimate represented by the second black dot, and so on. Figure 4 considerably exaggerates the process noise, since it is usually much smaller than the error bars. Tralli and Lichten [1990] and Lichten [1990b] describe this procedure more rigorously.

Over the period of 1 hour, optimal values for the random walk parameter allow for 0.5 to 2 cm variations in the zenith bias. Dixon and Kornreich Wolf [1990] investigated empirically the optimal value of this parameter, using daily
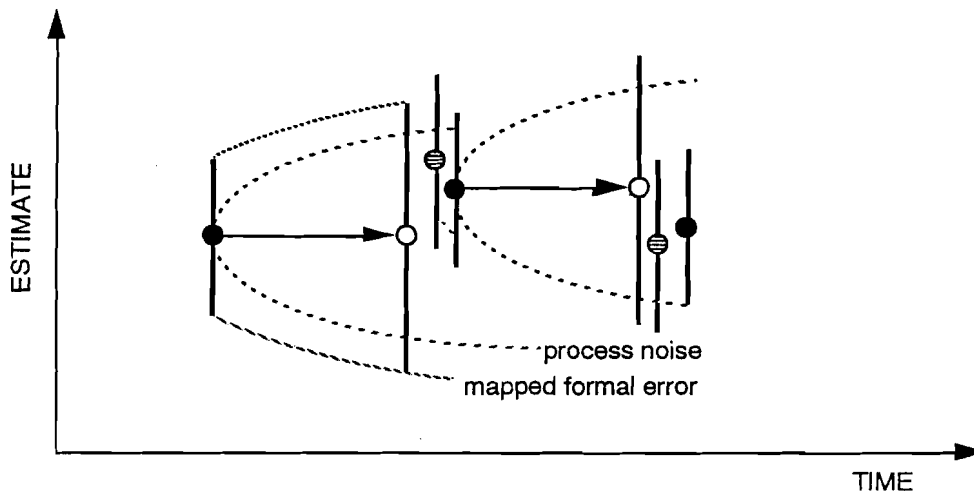

Fig. 4. An illustration of the application of random walk process noise.

baseline repeatability as an indicator; they found a value of 1.2 cm per square-root hour to give the best repeatability for the January 1988 GPS experiment in Central and South America (CASA Uno). Lichten and Border [1987], and Blewitt [1989] had used the same value with success in California. Introducing a novel way to assess estimation strategies, Dixon et al. [1991] optimized ambiguity resolution in the Caribbean region when using a random walk value of 1.0 cm per square-root hour. They also report that this value was consistent with observed variations (1.1 cm per square-root hour) in retrieved path delays from a WVR operating during the experiment.

Lichten and Border [1987] showed that estimating a stochastic, rather than a constant zenith tropospheric bias resulted in a factor of 2 improvement in baseline repeatability in California. They also reported a similar improvement in orbit repeatability (i.e., the RMS difference between independently determined orbit arcs, mapped to a common time interval). Tralli et al. [1988], analyzing data from the Gulf of California, Mexico, found that stochastic estimation of a zenith bias gave similar baseline repeatability as using WVR's. For the CASA Uno experiment, Dixon and Kornreich Wolf [1990] showed a factor of 2 improvement in baseline repeatability when the zenith tropospheric bias was estimated stochastically, giving almost identical results as using WVR's. Dixon et al. [1991] showed an insignificant difference in baseline repeatability in the Caribbean region when comparing the random-walk estimation method with the use of WVR's.

Local tropospheric modeling

For local networks (tens of km), tropospheric modeling and estimation techniques are essential to achieve few millimeter-level vertical precision. Gurtner et al. [1989] and Rothacher et al. [1990a] emphasized this point, especially for networks in mountainous regions. They suggested that statements concerning "millimeter-level precision" currently do not apply to the vertical components of baselines except to those shorter than 1 km where there is negligible height difference between the two receivers.

Local tropospheric models are important for rapid static surveying (to be discussed later), because there is not enough information in 5 minutes of data to adequately reduce the correlation between the zenith tropospheric delay and the vertical station coordinate. Employing a very simple local tropospheric model, Hurst et al. [1990] demonstrated 1.4 cm standard deviation in the vertical baseline component (for rapid static surveys separated by 3 months).

Stations separated by as much as 100 km have tropospheric delays that correlate to some extent. Davis et al. [1987] outlined a scheme that applies an a priori model of zenith tropospheric correlation as a function of baseline length, and demonstrated improved vertical repeatability. This method ought to be most useful when the information content of the data is deficient for some reason (e.g., when tracking only 4 satellites). Presumably, this method is not commonly used since most of today's geodetic receivers can track 8 satellites simultaneously.

Introduction

Ambiguity resolution refers to the determination of an integer-cycle bias for each set of double-difference carrier phase measurements. Ambiguity resolution is essential for the best achievable precision and accuracy, with improvement factors of 2 to 3 reported by Dong and Bock [1989] and Blewitt [1989], for regional-scale networks with baselines up to 2000 km.

In the early days of GPS, Counselman and Shapiro [1979] emphasized the importance of ambiguity resolution. Bossler et al. [1980] described the method whereby carrier phase biases are first estimated as real-valued parameters, then held fixed to integer values; hence the term "bias-fixing." Bias-fixing was routinely used in the early 1980's for short (few km) baselines. Bock et al. [1985a] demonstrated a full-network solution using bias-fixing. The network was tens of kilometers in aperture, and baseline precision was $10^{-6}L$. Such a network approach is now standard.

Long-Baseline Ambiguity Resolution

The problem of extending ambiguity resolution to longer baselines was not trivial, as evidenced by the 10 years that were to pass between papers by Counselman and Shapiro [1979] and the papers by Dong and Bock [1989] and Blewitt [1989], which describe the techniques in routine use today. The basic problems were (1) the paradox that, to be successful, the process of long baseline ambiguity resolution cannot proceed without first obtaining a precise geodetic solution, and (2) that, without additional information, the ambiguity parameters of the L1 and L2 phases are perfectly correlated.

The first problem requires (1) a sufficiently long arc of data (hours for regional scales), and (2) accurate modeling of the phase observables. For example, precise orbit determination is required for regional-scale ambiguity resolution. As we saw previously, Dixon et al. [1991] showed that ambiguity resolution improved by stochastically estimating the tropospheric delay. As baselines get longer, more models become important, such as tidal effects, and circular-polarization effects due to relative antenna rotation between the station and satellite [Wu, J.T. et al., 1993]. Accurate modeling of the phase observables does not require fiducial constraints, as Blewitt and Lichten [1992] pointed out with their demonstration of global-scale ambiguity resolution. The reason for this is that integer-cycle ambiguities are scalar quantities, which are not affected by choice of reference frame. In fact, ambiguity resolution may be impossible, or could have negative side effects, if the fiducial constraints have significant error.

The second problem, as Bender and Larden [1985] succinctly explained, can be solved by using pseudorange observations or ionospheric constraints (or both). One of their suggestions was using pseudorange data to estimate ionospheric delay, which could in turn be used to constrain the selection of possible integer values for the L1 and L2 channels. Melbourne

Fig. 5. Use of pseudorange for resolving the integer-cycle ambiguity in the widelane carrier phase data ($\phi1-\phi2$). The wavelength for widelane phase is 86.3 cm. This method is independent of baseline length [Melbourne, 1985].

[1985] and Wubbena [1985] independently proposed a more direct approach that uses pseudorange data to resolve the widelane ambiguity (referring to the phase difference between L1 and L2 channels). This method of widelaning, shown in Figure 5, has the useful property that it is model-independent, and therefore should not degrade with baseline length. Blewitt et al. [1988] and Dong and Bock [1989] applied this technique to resolve the widelane ambiguity (referring to the phase difference between L1 and L2 channels) on several baselines of ~1000 km. Without the availability of pseudorange data, the use of ionospheric constraints was implemented in various forms by Bock et al. [1986a], Shaffrin and Bock [1988], Dong and Bock [1989], and Blewitt [1989]. Abbot et al. [1989] demonstrated an alternative approach that explicitly estimates parameters of an ionospheric model.

Network Design and Bootstrapping

Around 1986, investigators agreed that a mix of baseline lengths enhances the ability to resolve integer-cycle ambiguities for regional geodesy. This was independently demonstrated by Blewitt et al. [1988], Dong and Bock [1989], and Counselman and Abbot [1989]. All these investigations used a ~4000 km-aperture fiducial network spanning North America with a broad mix of baseline lengths. There are several reasons why multiple baseline lengths assist ambiguity resolution: (1) longer baselines provide a wide-aperture view of the GPS orbits, reducing errors for the shorter baselines to much smaller than a wavelength; (2) if biases associated with the shortest baselines can be resolved, estimates of biases associated with longer baselines can be subsequently improved using the computed correlation between bias parameters; (3) biases associated with longer baselines can be expressed as a linear combination of several biases associated with intermediate baselines; thus, only biases associated with the intermediate baselines need be resolved.

Taking advantage of a wide distribution of baseline lengths in their networks, Dong and Bock [1989], Blewitt [1989], and Counselman and Abbot [1989] explained and demonstrated bootstrapping algorithms that were different in detail, but

similar in concept. As an example of such an approach: (1) select the best determined carrier phase bias first (where "best determined" is objectively defined as a function of the distance to the nearest integer, and the formal computed error); (2) if there is sufficient confidence, fix it to the nearest integer value, and compute how this perturbs all other carrier phase biases (which implicitly improves orbit parameters, station locations, etc.); (3) get the next best determined bias until a pre-specified confidence limit is exceeded; (4) back-substitute the fixed integer-bias solutions to recompute the geodetic and orbit estimates (retaining the unresolved biases as real-valued parameters). This algorithm uses off-diagonal information in the covariance matrix, but avoids the computationally prohibitive task of using all the information, which would require a search in a very high-dimensional lattice (and computation time longer than the age of the universe!). Key to such an approach is having short baselines (100 km) in the network to resolve ambiguities for the long baselines (~1000 km). Dong and Bock [1989] extended this approach by searching the lattice a few dimensions at a time.

Blewitt and Lichten [1992] showed that the above restrictions may be relaxed for a sufficiently dense global network. For a 21 station, global GPS network, they demonstrated ambiguity resolution without any bootstrapping for baselines up to 1500 km. Using bootstrapping then allowed the resolution of ambiguities for even the longest baselines (~10000 km).

TREATMENT OF CLOCK BIAS

The well-known double-difference method of eliminating clock bias, which has its origins before GPS [Counselman et al., 1972], allows for efficient computation of the least-squares geodetic solution. However, the clock elimination problem can be generalized to a clock estimation problem. Wu, J.T. [1984] describes a method of processing undifferenced data, which later evolved into the current method in the GIPSY/OASIS software [Wu, S.C. and Thornton, 1985] whereby clock biases are estimated as a stochastic process. "Stochastic estimation" here includes the logical extremes, ranging from the completely unpredictable behavior of white noise to models which allow time-correlated colored noise, polynomial behavior, or some combination of the above.

There are now several software packages [Swift, 1987; Lichten and Border, 1987; Landau, 1988; Andersen et al., 1992; Ch. Reigber, IGS electronic mail #40, 1992] which use the clock estimation method rather than the double-difference method, so it deserves some explanation as to the differences between these approaches.

First, let us discuss why differences between geodetic solutions may arise from softwares that implement different approaches to dealing with clock bias. What are the most significant causes of these differences?

Differences Between Stochastic Solutions

For a given stochastic model (e.g., white noise clocks), tests have shown that UD filtering versus SRIF filtering give

essentially identical results (limited by the numerical precision of the computer). This is perhaps not surprising; theoretically, UD and SRIF filters ought to give the same results despite the very different calculations which are carried out [Bierman, 1977]. Theoretically, Kalman filtering ought to give the same results too, however the numerical precision will be slightly degraded (because Kalman filtering deals with the covariance matrix instead of the square-root covariance matrix). This degradation is probably insignificant for double-precision machines, assuming an appropriate choice of units and nominal values.

Of course, changing the stochastic model will generally result in different solutions [Lichten and Border, 1987]. As the choice of stochastic model more truely represents the random behavior of the clock, so the solution will be more accurate. On the other hand, the quest for accuracy must consider the need for robustness. If the clock has an unpredictable glitch which is not represented by the stochastic model, then the constraints implicit in that model will unfavorably bias the solution. The white noise model, which assumes no correlation between past and future behavior, is the most robust model (it can readily tolerate step-functions in clock bias or rate, which are not uncommon). It is often said that white noise clock estimation is equivalent to double-differencing. As we shall see, this is not as simple as often stated, and in some cases the statement is false.

Finally, note that only relative clock behavior is estimable. Geodetic estimates can differ substantially due to an inappropriate selection of a reference clock. Unless the analyst corrects for the reference clocks errors using point positioning solutions, it is important to select a reference clock that is accurate to the few-microsecond level. (This corresponds to the time it takes for a GPS satellite to move approximately 1 cm.) Since many GPS receivers in the global network use H-maser external frequencies, this is not usually a problem.

Differences Between Double-Difference Solutions

For the double-difference method, answers can vary depending on the amount of rigor which is incorporated into the algorithm. The best software packages are quite rigorous, accounting for correlation in the double-difference observations, and implementing algorithms to select the double-difference observations which maximize information content. However, almost without exception, the actual practice of double-differencing begins with the formation of single-difference data files (between 2 stations), and it is at this stage that significant information can be lost if decisions are made by the user rather than by an automated, generalized algorithm.

Stochastic versus Double-Difference Methods

Phase and Pseudorange. One feature of stochastic estimation which will produce different solutions than double-differencing is the case where both carrier phase and pseudorange data are filtered simultaneously [Lichten and Border, 1987]. In this case, one common clock parameter can be estimated for both

carrier phase and pseudorange data, whereas double-differencing the pseudorange is equivalent to estimating an independent clock for that data type. In practice, this problem is addressed in double-differencing software by prior initialization of the clock parameters using pseudorange data in point positioning solutions. The results may be very effective, but in principle, a different least-squares problem is being solved.

Information Content. Ignoring the issue of simultaneous processing of carrier phase and pseudorange, let us return to the question, "Are double-differencing and white-noise clock estimation equivalent?" The answer is, "Sometimes." We must look at the underlying assumptions. For practical purposes, regional scale networks (or smaller) should give insignificantly different solutions, provided that the double-difference software rigorously deals with issues of correlation and data selection. However, it is a myth that they are theoretically equivalent for all data sets. They may, in fact, be different for global-scale data sets. Figure 6 illustrates a counter-example where no double-difference measurement can be constructed; however, the method of white noise clock estimation is able to extract geodetic information. In this example, 3 single-differences (from common satellites) could themselves be differenced to form a measurement which would eliminate clock bias. This is analogous to the double-difference measurement, where 2 single-differences are differenced. Double-differencing software cannot handle this case. However, it could be handled by what we might call "generalized differencing" software, an example being the Householder method proposed by Wu, J.T. [1984, 1986]. It would therefore be more correct to say that white noise clock estimation is equivalent to Householder clock-elimination rather than double-differencing. Of course, if the global network were sufficiently dense everywhere, the loss of information would be negligible. For most applications, this issue may not be worth serious consideration.

Practical Considerations. More relevant to most users are the practical consequences of choosing stochastic clock estimation versus double-differencing. Since double-differencing eliminates nuisance parameters up front, it is
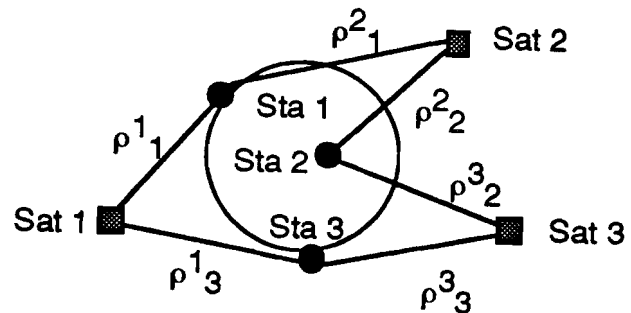

Fig. 6. In this example where each satellite is observed by 2 stations, no double-difference data can be formed; however the linear combination $\Delta = \left(\rho^1{}_1 - \rho^1{}_3\right) - \left(\rho^2{}_1 - \rho^2{}_2\right) - \left(\rho^3{}_2 - \rho^3{}_3\right)$ eliminates clock bias, and contains valuable geodetic information.

computationally more efficient. It is much easier to detect cycle slips in double-difference data at the preprocessing stage. This is a strong argument for using double-differencing software for codeless receivers without precise pseudorange capability. On the other hand, it is easy to find cycle-slips or data problems by inspecting post-fit residuals after filtering undifferenced data. In fact, this process can be automated, and the cycle-slips can be repaired by the same module that performs ambiguity resolution; but this is another reason why clock estimation can be more computationally expensive. However, these extra computations do provide the user with (1) residuals on a site-by-site basis, and (2) explicit, precise clock solutions for each receiver and satellite (relative to the selected reference clock) [Swift and Gouldman, 1989]. The RMS of post-fit residuals has been particularly useful as a measure of multipath at each of the global network sites.

EFFICIENT LOCAL SURVEYING METHODS

Ambiguity resolution techniques for kinematic, pseudokinematic, and rapid static surveying are quite different to methods used for long-baseline static positioning. First, let us define these surveying techniques. (As a note of caution, the following definitions may not be universally accepted.)

Kinematic surveying

Remondi [1985] developed kinematic surveying, which several investigators have used for precise positioning [Mader and Strange, 1987; Goad, 1989]. The kinematic technique uses a stationary reference GPS receiver, and another receiver that moves from point to point, roving within a radius of several kilometers. The analyst can resolve cycle ambiguities using an initialization technique (e.g., antenna swapping). Analysis software can recover from loss of phase lock, if there are enough satellites in view, or if the field operator, having realized that the receiver had lost lock, returned to the last stopping point. The field operator must therefore be quite careful, otherwise a large fraction of data could be rendered unuseable. Relative positioning accuracy at the 1-cm level is possible for site occupations of seconds [Ashkenazi et al., 1990], given at least 5 satellites should be in view, and provided that the environment does not cause significant multipath.

Pseudokinematic Surveying

Remondi [1988, 1990] also introduced pseudokinematic surveying. This method does not require that data be collected while moving an antenna from one point to another; however, it does require that each point be revisited a second time after a significant time-lapse (e.g., 1 hour). The idea is to connect phase between the two occupations, resulting in information content that is almost equivalent to staying on every point for the entire time-span.

The key to making this work is successful ambiguity resolution. The ambiguity function method, developed for VLBI phase analysis by Rogers et al. [1978], was first tested for GPS by Counselman and Gourevitch [1981]. This method

lay dormant until 1990 when Remondi [1990] and Mader [1990] revived it, successfully applying it to short baseline pseudokinematic surveying. Frei and Beutler [1990] have proposed and demonstrated the "fast ambiguity resolution method" (FARA), which searches the ambiguity lattice space for the most likely solution. In cases where return to the same site is unnecessary, the FARA method is applicable to rapid static surveying (see below) for codeless receivers.

Kinematic Positioning

Investigators have used kinematic positioning to track antennas on a moving platform (e.g., aircraft). In principle, the method is no different than kinematic surveying; the only difference is the application. The technique is described by Hatch [1986, 1989, 1990], Mader [1986, 1990], Evans [1986], and Hein et al. [1988]. This method has many applications; for example, monitoring sea-level by locating floating buoys relative to fixed points on land [Rocken et al., 1990]. Sea-floor geodesy as proposed by Spiess [1985] uses (1) acoustic ranging to locate a floating buoy relative to transponders fixed to the sea-floor, and (2) GPS to tie the buoy to points on land. Young et al. [1990] described a preliminary experiment showing the potential for marine geodesy to measure oceanic crustal motion. Minster and Genrich [1988] presented an approach to the simultaneous reduction of shipboard GPS data and gravity measurements.

Rapid Static Surveying

Rapid static surveying was a phrase introduced by Blewitt et al. [1989a] to describe a local surveying technique, which neither required that the receiver track while moving from one point to the next, nor required a second occupation of each site. Euler et al. [1990] described a similar technique, and Frei and Beutler [1990] proposed another successful approach. Typically a roving receiver stops at each site for about 5 to 10 minutes, and its location is referenced to a continuously operating receiver at a distance of up to 20 km or so. The time spent at each site could in principle be only a few seconds, but in practice, 5- to 10-minute occupations give the analyst enough data to assess the level multipath-induced errors at each site (an important error source for short data spans), and the occupation time is still less than the time it takes to travel between points, set up the equipment, and break it down.

Again, the trick is to be able to resolve the carrier phase ambiguities. The method of Blewitt et al. [1989b] is to use P-code pseudorange to resolve the widelane ambiguity, and then resolve the remaining ambiguity by relying on the likelihood that the differential L1–L2 delay between the fixed and roving receivers is much less than a wavelength (actually, it needs to be at the 1 cm-level to be sufficiently confident). During the 1989 peak of the 11-year solar cycle, it was possible to resolve ambiguities up to 30 km at night, but only over a few km during daylight hours. In 1992, ionospheric activity had reduced such that it became possible to resolve ambiguities over 20 km during daylight hours. The years of 1993-1996 should prove to be very favorable to the rapid static technique

during daylight hours. (Ionospheric modeling may be useful to extend the application of rapid static surveying, but it is difficult to account for fine structure at short temporal and spatial scales [Abidin and Wells, 1990].)

Having surveyed the network for the first time, subsequent surveys could use codeless receivers, since ambiguity resolution can now proceed by applying geodetic constraints from the first solution (provided that station motion in the intervening time is only a few centimeters). Once the ambiguities are resolved, the solution can be recomputed such that it is not biased by the first solution. Blewitt et al. [1989b] used rapid static surveying to monitor post-seismic motion following the Loma Prieta earthquake of October 1989. Hurst et al. [1990] showed that inter-experiment repeatability was sub centimeter in horizontal components, and 1.4 cm in the vertical over ~20 km (see above section on tropospheric modeling).

ASSESSMENT OF GEODETIC ACCURACY AND PRECISION

For a detailed assessment of GPS precision and accuracy over regional scales, see the in-depth study by Larson and Agnew [1991] who used ~2.5 years of data from California. Larson et al. [1991] went further into the question of systematic errors introduced by constraining the coordinates of fiducial sites.

Three methods have been widely used for assessing GPS geodetic accuracy and precision: (1) repeatability (i.e., weighted RMS scatter) of coordinate estimates, (2) intercomparison with other techniques, and (3) formal computation of the expected standard deviations using the measurement matrix and an assumed data variance. All three of these methods have yielded reasonably comparable results, provided there are no data problems and the estimation strategy is appropriate. In producing error bars for coordinate estimates, investigators have frequently used either methods (1) or (3), or some combination of the two (for example, scaling the formal covariance such that computed baseline length errors agree on average with the repeatability).

Daily Repeatability

Baseline repeatability is a function of the time spanned by each solution. Typical regional experiments last several days, in which case daily repeatability is a valuable indicator of precision [Bock et al., 1985a]. The improvement in daily repeatability is perhaps the best indicator of successful ambiguity resolution; an improvement by a factor of 2 to 3 is typical for static surveying [Bock et al., 1985a; Dong and Bock, 1989; Blewitt, 1989]. Repeatability has a well-defined relationship with baseline length only if fiducial constraints are not used. When reporting repeatability as a function of length, it is important not to draw baselines between stations that are near fiducial stations. In this case, length is not a relevant characteristic of the baseline; more relevant is the distance to the nearest fiducial station.

A comprehensive study of California GPS solutions by Larson and Agnew [1991] shows daily baseline repeatability of 2 mm + 6 × $10^{-9}L$ for the north, 2 mm + 13 × $10^{-9}L$ for the

east, and 17 mm for the vertical component, with no obvious length dependence. The limiting repeatability of 2 to 4 mm observed by many investigators for short (but not very short) baselines is probably due to tropospheric variations (since multipath errors approximately repeat from day to day).

Results from ~10-m baselines show daily repeatability better than 0.1 mm (George Purcell, private comm.). Genrich and Bock [1992] used GPS as a strain meter across the San Andreas Fault, showing sub-millimeter daily repeatability for ~100 meter baselines with short occupation times. They employed a method to filter out the daily repeating multipath signals. Such a technique has applications to kinematic, pseudokinematic, and rapid static surveys.

Long-Term Repeatability

Many systematic errors (e.g., tropospheric mapping function errors, mismodeling of solar radiation pressure, or multipathing) tend to repeat from day to day; thus, long-term repeatability is a much better indication of precision. Of course, if results are to be compared over several years, the results should be detrended to remove secular crustal motions. This approach has been taken by Murray et al. [1988], Davis et al. [1989], Prescott et al. [1989], Larson and Agnew [1991], and Murray [1991]. These studies typically indicate long-term regional baseline repeatability (up to ~400 km) at the several-millimeter level in the north and east baseline components, and 20-40 millimeters in the vertical component. Of course, these early studies have necessarily included data from 4-channel receivers, so we should expect recent solutions using 8-channel receivers to prove to be more precise than this.

The vertical baseline component generally has a greater standard deviation than horizontal components. This is simply because we can only look up at the satellites, and not down; hence the data do not constrain the vertical as well, and the vertical is more sensitive to errors in tropospheric delay. Repeatability in the vertical of ~30 mm was quite common using 4-channel receivers. Using 8-channel receivers in the California Permanent GPS Geodetic Array (PGGA), Lindqwister et al. [1991] showed sub centimeter repeatability in the vertical component for regional-scale baselines. This improvement in precision partly results from the dramatic reduction in the magnitude of the correlation coefficient between the vertical parameter and the zenith troposphere parameter when increasing the number of simultaneously observed satellites from 4 to 5 (and to a lesser extent from 5 to 6, etc.).

As we saw in Figure 2, Bock et al. [1993] and Blewitt et al. [1993] detected a co-seismic displacement of a few millimeters on a 1700 km baseline (from JPLM, California to DRAO, British Columbia), caused by the Landers earthquake sequence of June 28, 1992. Each group independently (and with different software) estimated absolute co-seismic displacement estimates for a few stations in California. These estimates agreed to within their 95% confidence ellipses. For PGGA baselines (~200 km), the daily baseline repeatability (over a

period of 2 months) was 4 mm (horizontal) and 10–15 mm (vertical).

Intercomparisons

Although repeatability is an important tool to assess solution precision, it is clearly important to assess accuracy by comparing GPS results with the VLBI and satellite laser ranging (SLR) techniques. This is not trivial, since one must correctly account for the vector between the phase center of the GPS antenna and the intersection of axes of the VLBI antenna (and similarly for SLR). This vector comprises of (1) a test-range measurement of the vector between the GPS phase center and a reference mark on the antenna, (2) the eccentricity vector of the antenna relative to the underlying monument, (3) the vector between the GPS monument and the monument under the VLBI antenna, and (4) the vector between this monument and intersection of the VLBI antenna's rotation axes (information that the original engineering diagrams may contain). Moreover, one must also account for the crustal motion between, the experiment epoch, and the quoted reference epoch of the VLBI solution.

Despite these difficulties, there have been several successful intercomparisons between coordinates estimated by SLR, VLBI, and GPS [Ray et al., 1991; Blewitt et al., 1992], and this method remains perhaps the most believable indicator (albeit a conservative one) of the accuracy of all three techniques. Intercomparisons by Beutler et al. [1987] showed few-centimeter agreement in Alaska. Strange [1988] showed centimeter-level agreement on several baselines in California. Davis et al. [1989] report repeated measurements of the Pales Verdes-Mojave-Vandenberg triangle, indicating centimeter-level agreement. Lichten and Bertiger [1989] demonstrated $1 \times 10^{-8}L$ accuracy for $L > 1000$ km. Larson [1990] used the technique of comparing GPS baseline rate with VLBI, which has the advantage that it is independent of ground survey errors. A linear fit to 18 GPS occupations of the Mojave-Vandenberg baseline (over a span of 2.3 years) gave agreement at the level of 2 mm/yr.

GLOBAL GEODETIC PRECISION

By 1989, regional baseline precision had clearly become limited by orbit accuracy, which in turn was limited by the size of the satellite tracking networks. During the late 1980's a global network of GPS receivers gradually came into existence. Called "CIGNET" (Cooperative International GPS NETwork), this saved the investigators the burden of installing tracking receivers at fiducial points every time they conducted an experiment. CIGNET has been used to advantage for regional GPS investigations all over the globe (for example, by university researchers collaborating in the UNAVCO consortium). The National Geodetic Survey (NGS) continues to play a central role in CIGNET in the retrieval and dissemination of global GPS tracking data to the geodetic community.

A temporary, semi-global tracking network supported the January 1988, "CASA UNO" experiment in Central and South America. Results by several investigators showed the tangible improvement in regional baseline precision due to near-global tracking [Kornreich Wolf et al., 1990; Freymueller and Kellogg, 1990; Lichten 1990a]. Focusing on the global tracking network itself, Schutz et al. [1990] showed $1 \times 10^{-8}L$ baseline precision over thousands of kilometers.

Following the CASA UNO experiment, Melbourne et al. [1988] and Mueller [1990] proposed that an international organization install and maintain a global high-quality GPS network to provide a service to geodynamics investigators, allowing for routine baseline precision of a few $\times 10^{-9}L$. As a test campaign under the auspices of the International Earth Rotation Service (IERS), the GIG'91 experiment successfully demonstrated precise global tracking during 3 weeks in early 1991 [Blewitt, 1991]. GIG'91 served as a prototype to the global network under development today. Using the GIPSY/OASIS software to resolve ambiguities on a global scale, Blewitt and Lichten [1992] showed baseline repeatability of 2 mm + $2 \times 10^{-9}L$ from 21 globally distributed P-code receivers, without the use of fiducial constraints. Blewitt et al. [1992] transformed a global network solution into the International Terrestrial Reference Frame (ITRF) [Boucher and Altamimi, 1991], showing 12 mm RMS agreement for 33 station coordinates in the northern hemisphere. Using the GEOSAT software to analyze the GIG'91 data, Andersen et al. [1992] demonstrated baseline repeatability of 1 mm + $2 \times 10^{-9}L$ for baselines shorter than 4000 km, and, after transforming the solution to ITRF, showed 15 mm RMS agreement with ITRF coordinates.

The results from the GIG'91 experiment represented a major advance in demonstrating the potential of GPS, giving precision at least an order of magnitude better for UT1 variations, polar motion, and geocenter determination than earlier pioneering work by Swift [1985, 1989], Bangert [1986], Abbot et al. [1988], Malys [1988], and Malla and Wu [1989]. The estimated location of the Earth center of mass was within 15 cm of the ITRF geocenter [Vigue et al., 1992]. Herring et al. [1991] and Lindqwister et al. [1992] showed 0.5 marcsec (< 2 cm) agreement between Earth pole orientation variations estimated by GPS, VLBI, and SLR. Lichten et al. [1992] showed 50 μsec (2 cm) agreement in UT1 variations when comparing GPS to VLBI.

Both (1) the impressive results from the 3-week GIG'91 experiment, and (2) the convenience that CIGNET provided, were important factors in convincing the international community that the time had come for an operational "International GPS and Geodynamics Service" (IGS) [Mueller, 1990]. Several independent groups analyzed data from 20 to 30 stations every day during the first 3 months of full-time IGS operations (which began June 1992), their results indicating sub-meter level GPS orbit accuracy, marcsec-level daily estimates of the Earth pole direction, and centimeter-level determination of global station coordinates.

It is important to realize that the global network is now sufficiently dense that geocentric station coordinates (latitude, longitude, and geodetic height) can be estimated with sub
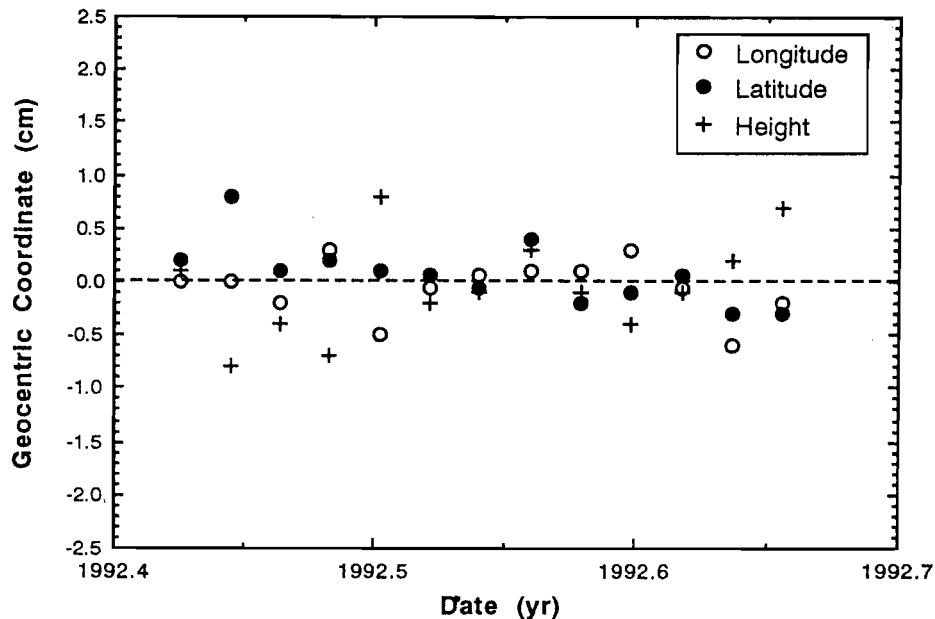
Fig. 7. Time-series of weekly geocentric coordinate estimates for the GPS global network station at Wettzell, Germany, from 1 June, 1992 to 30 August, 1992.

centimeter precision. As an example of this, Figure 7 shows a time series of weekly geocentric coordinate solutions (minus nominal values) for Wettzell, Germany, from 1 June to 30 August, 1992 (during the IGS test campaign). The global network solution, from which Figure 7 was derived, is a pure GPS solution (using GPS estimates of polar motion and length of day, and using no fiducials). The level of height precision demonstrated appears to be sufficient to meet the requirements (few mm per year) for measuring post-glacial rebound over a period of several years. It also demonstrates the role that GPS could play in measuring global sea-level change. It is worth noting that the solutions presented in Figure 7 make no use of surface meterological data or WVR data.

Software Development

General Software Features

Concurrent with the above demonstration of techniques was the development of GPS geodetic data analysis software that embodied algorithms as they became established. Beutler et al. [1988] point out that software designers must consider (1) the end user, (2) the technical problems to be addressed, and (3) the definition of guidelines on structuring the system. Following this outline, let us first recognize that the kind of processing software we are discussing here is for scientific use and professional use in agencies concerned with high-precision surveys. Second, software packages need to include (1) accurate orbit integration with appropriate force models; (2) data editing capability; (3) accurate modeling of Earth orientation, tides, and media propagation; (4) capability to estimate station coordinates, satellite states, tropospheric bias,

receiver time-tag errors, carrier phase biases, UT1 variations, and polar motion; (5) an algorithm implementing the bootstrapping mechanism for ambiguity resolution.

The adoption of the RINEX standard format [Gurtner and Mader, 1990] largely solved the problem of handling different receiver data formats; however, for some mixed-receiver networks, normal pointing algorithms must align receiver time-tags to the same epochs to eliminate clock bias [Blewitt, 1990c]. In doing so, the algorithm must be careful to reduce systematic error from frequency variations caused by selective availability [Wu, S.C., et al., 1990; Davis, E.S., 1990; Feigl et al., 1990; Rocken and Meertens, 1991].

All software packages must be capable of handling outliers and cycle-slips in the carrier phase data. As mentioned previously, software packages that form double-differences are probably better at handling codeless data. This is an important consideration for users which process data from a wide variety of receiver types. For the reduction of global-scale data sets, the user should consider processing undifferenced data from receivers with high-precision pseudorange (as discussed previously). Useful products of undifferenced data processing include (1) post-fit residuals for individual station-satellite pairs, and (2) explicit clock estimates. The choice of double-differencing versus stochastic estimation is not simply a question of which is better, but which better suits the needs of the user.

Another user consideration is "Sequential filter or standard inversion of the normal equations?". A sequential filter (e.g., Kalman, UD, or square-root information) is useful to fit variations in tropospheric delay and unmodeled satellite forces [S.C. Wu and Thornton, 1985; Lichten, 1990b]. However,

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experience from VLBI seems to imply that piecewise linear models for the troposphere may be as effective as random walk stochastic estimation in improving long-term baseline repeatability.

"Should the software be more interactively oriented, or batch oriented?" For routine analysis of permanent GPS arrays, batch processing is obviously preferable, with the entire processing as automated as possible. Interactively oriented software is much better for users who conduct field experiments, where new sites are the rule rather than the exception. A third consideration is the flexibility of the software, which allows the user to perform R&D style analyses, testing out new Earth models, orbit dynamics, and estimation strategies. (Rarely is this type of software available commercially.)

It is quite a common practice for the users of today's high precision software to contribute to its improvement. Modularity is essential in this regard. It should be possible for a software developer to simply insert a new data editing module into the overall data flow. What is not a universal feature of today's software is the ability for the non-specialist to produce a high-precision geodetic solution reliably on an arbitrary data set without a significant amount of training (or luck). However, as stated previously, we are discussing software that is designed for professional use in agencies concerned with high-accuracy surveys, and training is usually facilitated by such agencies. As systems are becoming more precise, robust, and automated, and as distributed ephemerides are becoming more accurate and reliable, geophysicists should soon be able to produce good solutions without much training other than reading a short user's guide.

Major Software Packages

The following is an alphabetically ordered list of major software packages, which are in routine use today for geodynamics research, are capable of orbit improvement, and are capable of producing $10^{-8}L$ baseline accuracy.

BERNESE by the University of Berne, Switzerland [Gurtner et al., 1985; Beutler et al., 1987, 1988; Rothacher et al., 1990b; Davis et al., 1989] is available commercially. This is probably the most widely used package by the geodynamics community. It is used for daily IGS analysis. It forms double-difference data.

DIPOP-E by the University of New Brunswick, Canada [Chen and Langley, 1990] forms double-difference data.

GAMIT by the Massachusetts Institute of Technology [Bock et al., 1986a; Schaffrin and Bock, 1988; Dong and Bock, 1989]. The recent combination with the Kalman filter in the GLOBK software allows for stochastic estimation of orbit parameters [Herring et al., 1991]. It is used for daily IGS analysis. It forms double-difference data.

GEODYN [Martin, 1985] and its commercial derivative MicroCosm [Van Martin Systems Inc., 1990; Colombo, 1989; and Ferland and Lahaye, 1990] developed by NASA Goddard Space Flight Center. It forms double-difference data.

GEOSAT by the Norwegian Defense Research Establishment

[Andersen, 1986; Andersen et al., 1992]. The most recent version uses a UD factorized filter instead of double-differencing.

GEPHARD4.0 by the GeoForschungsZentrum, Potsdam, Germany [Ch. Reigber, IGS electronic mail #40, 1992]. It is used for daily IGS analysis. It uses a filter instead of double-differencing.

GIPSY/OASIS by the Jet Propulsion Laboratory [Sovers and Border, 1987; Lichten and Border, 1987; Blewitt, 1989, 1990a; Lichten, 1990b]. The new GIPSY/OASIS II is currently used for daily IGS analysis, uses a SRIF filter, and can estimate stochastic satellite forces.

GPSOBS/BAHN5 by the European Space Agency, Darmstadt, Germany [J. Dow, IGS electronic mail #51, 1992]. It is used for daily IGS analysis. It forms double-difference data.

MSODP1/TEXGAP by the University of Texas at Austin [Abusali et al., 1986; Schutz et al., 1989, 1990]. It is currently used for daily IGS analysis. It uses double-differencing.

TOPAS by the Institute of Astronomical and Physical Geodesy, University of FAF, Munich, Germany [Landau, 1988; Bastos et al., 1990] is available commercially. It uses a UD factorized filter.

LIMITS ON GPS-GEODETIC PRECISION

Long Baseline Precision

GPS-baseline repeatability is a function of baseline length. Using a sample of 136 baselines over 21 days, Heflin et al. [1992] demonstrated that this function is almost perfectly linear for global-scale free-network solutions. Presumably, this trend is due to error sources that decorrelate with increasing baseline length. Since VLBI analyses show a similar trend [Caprette et al., 1990, p. 24], we cannot assume that GPS-orbit errors dominate (as is often assumed). Perhaps the most important source of error is tropospheric delay, which, through the curvature of the Earth, increasingly maps into baseline length errors for longer baselines.

Goal for Estimating Tropospheric Delay

Each station's zenith tropospheric delay is almost completely decorrelated from neighboring stations over 1000 km away, causing random errors for each station's position in the vertical direction. The following simple derivation (which assumes a spherical Earth) demonstrates that this produces a scaling law. From Figure 8, we see that an error in the vertical ΔV at one end of a baseline causes an error in length $\Delta L \approx \Delta VL/2R$, where L is the baseline length, and R is the earth radius.

If we assume that tropospheric errors are uncorrelated for global networks (with baselines > 1000 km), we get the following scaling law:

$$\frac{\sigma_L}{L} = \frac{\sigma_V}{\sqrt{2}R} \tag{1}$$

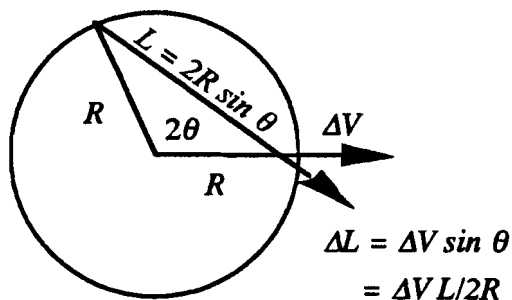where σ_V is the standard deviation in the vertical station

Fig. 8. Scaling of baseline length error caused by vertical errors.

coordinate, and σ_L is the standard deviation in baseline length, which is a linear function of baseline length provided σ_V is independent of baseline length. Equation (1) shows that σ_V must be less than 9 mm to achieve baseline precision better than $10^{-9}L$. Since errors in σ_V correspond roughly to 3-4 times the error in wet zenith tropospheric delay, this requires 3-mm accuracy in the zenith tropospheric delay. Water vapor radiometers have not yet consistently demonstrated performance at this level. This goal may be achievable using the GPS data themselves to estimate tropospheric delay, using mapping functions perhaps tailored to specific sites, to the time of year, and to meteorological conditions. Current accuracy is probably at the 10 mm level.

Local and Regional Scales.

We can use the above information to construct a limiting error model for the vertical component of local and regional scale baselines. Tropospheric delay is very correlated on a local scale, and is to some extent correlated on a regional scale, so we expect the scaling law to break down somewhere below 1000 km. If we assume that Kolmogorov turbulence provides a reasonable model for tropospheric variations, we would expect the variance in differential tropospheric delay to be proportional to $L^{2/3}$ [Truehaft and Lanyi, 1987]. Turbulence can be described as an ever increasing size of cells, thus implying some level of correlation in tropospheric delay even for the longest baselines; however, in reality other factors (such as the presence of mountain ranges and coastlines) may limit the validity of this law to regional scales (or even shorter). For purposes of discussion, let us assume (1) this scaling law is correct, (2) we attempt to use this information in some optimal estimation strategy, and (3) final errors in the vertical baseline coordinates are proportional to the RMS variations in differential tropospheric delay:

$$\sigma_V = 3\varepsilon\alpha L^{1/3} \tag{2}$$

where the factor of 3 roughly accounts for the mapping of zenith tropospheric errors into vertical errors, ε is the fraction of the wet delay that is incorrectly modeled, and the theoretical work of Truehaft and Lanyi [1987] suggests a typical value $\alpha = 5 \times 10^{-6}$ km$^{2/3}$. Let us further assume that we can achieve the

above goal and estimate the absolute zenith troposphere with an accuracy of 3 mm. By substituting $L = 1000$ km in equation (2), this corresponds roughly to a value $\varepsilon = 0.05$; that is, the wet troposphere is modeled with 5% accuracy. Thus, in the absence of other error sources (such as multipathing), the limiting vertical error on regional scales is

$$\sigma_V \text{ (mm)} = 0.75L^{1/3}\text{(km)} \tag{3}$$

This gives values of 0.35 mm over 0.1 km, 0.75 mm over 1 km, 1.6 mm over 10 km, 3.5 mm over 100 km, 7.5 mm over 1000 km. Again, it must be emphasized that results of this quality would require estimation of the wet troposphere to better than 5%, that multipath is adequately controlled or calibrated, and that orbit mismodeling is negligible.

This model agrees quite well with daily vertical repeatability by Genrich and Bock [1992] over 0.1-1 km where the data were calibrated for multipath. Daily vertical repeatability at the 4-8 mm level was observed by Lindqwister et al. [1991] over distances of 100-200 km, where zenith tropospheric delay was stochastically estimated using a random walk constraint of 12 mm per square-root hour. For their study, multipath probably caused a vertical bias which repeated almost exactly from day to day. Of course, this comparison in itself does not validate the model; the main purpose of this exercise was to determine a rough criterion as to how well we must estimate wet tropospheric delay in order to satisfy geodetic goals, and the answer appears to be < 5%.

Baseline length error on local and regional scales are probably limited by multipathing errors and azimuthal asymmetry in tropospheric delay. Limiting orbital errors would be almost insignificant (< 1 mm baseline length error) for baselines shorter than 1000 km. There is therefore reason for optimism for achieving millimeter-level precision in the horizontal components of regional baselines.

Future Developments

Future technological developments will be aimed at (1) automating data analysis, allowing for routine global network estimates, and (2) reducing error sources, such as tropospheric delay as discussed above. Promising work in filtering out multipath error from day to day has been demonstrated by Genrich and Bock [1992]. Higher order ionospheric effects may become significant for baselines longer than 1000 km (approximately the correlation distance of the ionosphere). Work on reducing higher order ionospheric effects is presented by Brunner and Gu [1991]. Other areas that require more work include orbit modeling (especially for eclipsing satellites); global ambiguity resolution [Blewitt and Lichten, 1992]; antennas [Schupler and Clark, 1991]; mixing receiver types [Rocken and Meertens, unpublished UNAVCO report, 1992], allowing for anisotropic variations and perhaps including auxiliary meteorological information to tune mapping functions and filtering strategies; filtering strategies for estimating UT1 and polar motion; and improved models for Earth tides, and oceanic and atmospheric loading.

CONCLUSIONS

The last decade has witnessed roughly 3 orders of magnitude improvement in GPS-baseline precision over all distance scales. Recent reports indicate the best RMS precision ranges from 0.1 mm on a very local scale (~10 m), to 1 cm on a global scale (~10,000 km). Daily changes in the orientation of the Earth's pole and the rotation of the Earth's crust about the pole can be measured with a precision of ~15 mm at the Earth's surface. Advances in receiver hardware, data analysis software, and analysis techniques have made these results possible. Further improvements to present systems are possible, the most pressing being the ability to produce high-quality results routinely and with less effort. At the time of writing, this issue was a major concern for the success of the IGS. However, since June 1992, several international groups have succeeded in producing highly precise GPS global solutions every day. The emphasis will now probably revert to further improvements in media delay, multipathing, antenna phase center variation, the modeling of satellite forces, Earth tidal and loading effects, Earth orientation estimation strategy, higher order ionospheric delay, and routine ambiguity resolution on a global scale.

It is likely that, within the next few years, ~200 permanently located, high-precision GPS receivers will be operating continuously for geodynamics research. In some respects, the challenge of handling this much data is not as great as the initial hurdle of routinely reducing the data from the current ~30 station global network. Obviously, partitioning will play a large role in reducing the least-squares problem to a practical level. What will be a challenge is to combine the various regional solutions into a single, consistent time-series, which shows daily variations in the shape, orientation, scale, and origin of the global polyhedron. The Earth will be approximated by a 200-vertex, rotating, deforming, polyhedron, with important details filled in by regional and local experiments.

Ending on a personal note, I believe this development represents an inevitable new paradigm for geodesy, which will influence the way we think about the Earth and the meaning of geodesy. The development of GPS technology has not been the only responsible factor; true international cooperation and the vision of the geodetic community have been just as important.

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