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GLOBAL COORDINATES WITH CENTIMETER ACCURACY IN THE INTERNATIONAL TERRESTRIAL REFERENCE FRAME USING GPS

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Abstract. Using 21 days of Global Positioning System (GPS) data from 21 globally distributed receivers operating during early 1991, we solve for a 7-parameter transformation between a GPS free-network solution and coordinates of 12 stations listed in the International Terrestrial Reference Frame (ITRF). Standard errors of GPS coordinates are derived by applying an orthogonal projection operator to the free-network covariance. The weighted RMS difference between 33 transformed GPS and ITRF coordinates is 12 mm in the northern hemisphere. Best results are obtained by mapping ITRF coordinates to the epoch of this experiment assuming no vertical site motions. Fixing selected sites in the GPS solution to ITRF'90 does not improve the agreement. We conclude the use of fiducial constraints is unnecessary for global networks.

Introduction

In order to use GPS for monitoring large-scale geophysical signals such as global sea-level change and post-glacial crustal rebound, GPS must be capable of providing global station coordinates with centimeter-level accuracy. The "fiducial concept," in which a subset of coordinates are constrained to values previously determined by other space-based techniques, has been a successful GPS technique over regional scales. Use of the fiducial concept implies that such solutions are no longer independent of other techniques; fiducial coordinate errors can bias the GPS coordinates and even strain-rates [*Larson et al.*, 1991]. Moreover, on a global scale it becomes difficult to interpret GPS solutions which are strongly constrained a priori. In this paper, we show that (1) high-precision GPS coordinates can be derived without use of fiducial constraints, and (2) fiducial constraints do not improve the accuracy of our global network solution.

Heflin et al. [1992] applied the free-network approach in an analysis of 21 days of data from 21 globally distributed receivers in the GIG'91 campaign beginning January 22, 1991. Expanding on this, we assess coordinate accuracy by performing a similarity transformation of this network into the International Terrestrial Reference Frame, ITRF'90. The utility of fiducial constraints for this global network is tested.

Data Used in This Study

For this study, we use a free-network GPS solution, conveniently represented by Cartesian site coordinates, very loosely constrained to the ITRF. We refer the reader to *Heflin et al.* [1992] for details of the data reduction. In this study, the Earth's pole position and satellite orbits were estimated independently every 24 hours to reduce systematic errors.

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Of the 21 sites in this GPS solution, we have obtained local surveys from the GPS antenna to an ITRF monument for 13 "collocated" sites. (These surveys are completely independent of the GPS solution used in this study). ITRF'90 coordinates and standard deviations at epoch 1988.0 are given by *IERS* [1991]. ITRF'90 represents a combination of solutions by various groups using very long baseline interferometry (VLBI) or Satellite Laser Ranging (SLR). Since the coordinates for YELLOWKNIFE have large error bars in ITRF'90, this site is excluded from the intercomparison. The remaining 36 coordinates have standard deviations no larger than 2.1 cm. Local survey solutions are listed in Table 1 for the GPS monuments in our solution.

Site Velocity Models

To account for tectonic motion between ITRF'90 reference epoch 1988.0 and epoch 1991.1 we apply two models. Model A uses empirical site velocities from solution GLB718 for VLBI sites [*C. Ma, D.S. Caprette, and J.W. Ryan*, private communication, 1991], and solution SSC-(IERS)-90C01 for SLR sites [*Boucher and Altamimi,* 1991]. Model B is the same, except that the 4 collocated sites which have vertical velocities > 1 mm/yr in GLB718 (e.g., MADRID at

TABLE 1. Local Survey Solutions (GPS - ITRF'90)

SITE NAME	DELTA COORDINATE (mm) X	Y	Z	ANTENNA* HEIGHT (mm)
ALGONQUIN	94763	61017	6666	184
FAIRBANKS	-74141	49280	-31231	166
GOLDSTONE	2879994	5222210	8549901	0
PASADENA	1987	-18063	-22052	163
KAUAI	7941	-23617	-4407	163
KOOTWIJK	-12461	-37503	23024	156
MADRID	-134193	159660	164320	0
MATERA	-14128	-13229	18027	352
PINYON	125588	117655	279910	1914
TROMSO	36288	-33115	-9219	2543
WETTZELL	39384	71978	-58362	1668
YARRAGADEE	-18605	-12471	-5835	143
YELLOWKNIFE	-327848	314599	82882	181
CANBERRA	*(not yet available)*			0
JOHANNESBURG	*(not yet available)*			9754
HONEFOSS	*(not in ITRF'90)*			4017
NY ALLESUND	*(not in ITRF'90)*			1810
SIDNEY BC			*Until Jan 28:*	3360
			From Jan 29:	1349
SANTIAGO	*(not yet available)*			183
SCRIPPS	*(not in ITRF'90)*			1915
USUDA	*(not in ITRF'90)*			0

*GPS antenna height is to the top of the Rogue choke ring. An additional phase center height of -20.7 mm was assumed for the dual-frequency combined phase and pseudorange observables. These antenna heights may not be valid for dates other than January 22 - February 13, 1991.

TABLE 2. ITRF'90 Coordinates of GPS Monuments at Epoch 1991.1

SITE		COORDINATE AT EPOCH 1991.1 (mm)			VELOCITY (mm/yr)			ERROR (mm) **		
NAME	MODEL	X	Y	Z	X	Y	Z	X	Y	Z
ALGO		918129630	-4346071223	4561977794	-18.7	-3.6	1.1	7	8	9
FAIR		-2281621298	-1453595751	5756961960	-24.1	-0.1	-10.3	8	9	9
GOLD	A	-2353614080	-4641385442	3676976512	-17.4	11.1	-9.9	8	10	9
	B*	-2353614082	-4641385457	3676976523	-18.5	5.3	-5.2	8	10	9
PASA	A	-2493304032	-4655215589	3565497327	-35.2	23.9	3.4	13	18	14
	B*	-2493304035	-4655215595	3565497331	-35.9	22.6	4.4	13	18	14
KAUA		-5543838061	-2054587585	2387809490	-13.1	69.8	29.7	9	9	10
KOOT		3899225380	396731754	5015078290	-15.3	16.0	10.5	18	19	17
MADR	A	4849202647	-360329222	4114913118	17.9	18.9	44.2	15	10	17
	B*	4849202592	-360329223	4114913058	-10.2	18.8	13.6	15	10	17
MATE		4641950906	1393056751	4133280313	-16.2	18.1	12.1	11	11	11
PINY	A	-2369510349	-4761207198	3511396053	-25.7	23.7	-3.4	11	15	13
	B*	-2369510362	-4761207226	3511396075	-29.3	16.1	2.1	11	15	13
TROM		2102940468	721569359	5958192044	-19.0	10.4	5.4	14	12	21
WETT		4075579405	931807155	4801570931	-14.4	12.5	9.5	9	9	9
YARR		-2389025280	5043316823	-3078530992	-51.5	8.0	53.0	12	12	13

*Model B has both a different velocity and epoch value at 1988.0. In the case of Goldstone, the velocity solution for station MOJAVE12(7222) was applied from GLB718.
**Errors as published in *IERS* [1991] are mostly underestimated since they refer to 1988.0

41 mm/yr) were mapped to 1991.1 assuming no vertical motion. Since ITRF'90 implicitly contains GLB718, we were able to account for the correlation between 1988.0 site coordinates and velocity components in mapping coordinates to 1991.1. Table 2 lists ITRF'90 coordinates mapped to GPS monuments at epoch 1991.1 for both velocity models.

Similarity Transformation Model

The similarity transformation used by IERS is:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix} + \begin{pmatrix} s & -\theta_z & \theta_y \\ \theta_z & s & -\theta_x \\ -\theta_y & \theta_x & s \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \qquad (1)$$

where x, y, and z are GPS-derived coordinates; X, Y, and Z are ITRF'90 coordinates; t_x, t_y, and t_z represent an offset in origin; s represents a difference in scale; θ_x, θ_y, and θ_z represent differences in orientation. Some parameters have physical interpretations: (1) the offset in origin represents a discrepancy in the estimated location of the Earth center of mass and should be statistically consistent with zero; (2) since both systems adopt the same speed of light, gravitational coefficient $GM_\oplus$, and definition of coordinate time, the scale difference should be consistent with zero; (3) orientation differences have no physical significance, since they are arbitrarily constrained by loose a priori variances.

For purposes of estimating the transformation parameters, equation (1) can be rearranged and rewritten in matrix form:

$$\delta \mathbf{x} = \mathbf{A}\beta + \nu \qquad (2)$$

where $\delta\mathbf{x}$ is the vector of coordinate differences (GPS-ITRF), β is the vector of 7 tranformation parameters, $\mathbf{A}$ is the matrix of partial derivatives, and a vector of errors ν has been included. The weighted least-squares estimate of β is

$$\hat{\beta} = (\mathbf{A}^T\mathbf{W}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{W}\,\delta\mathbf{x} \qquad (3)$$

where $\mathbf{W}$ is an assumed weight matrix designed to reflect the relative strength of the various input coordinate data.

Constructing the Weight Matrix

We do not make the usual choice of weights $\mathbf{W}=\Sigma^{-1}$ where Σ is the initial GPS covariance, because Σ is only saved from being non-singular by loose a priori constraints. This arises from (1) perfect correlation between the satellite ascending nodes and the origin of longitudes, and (2) perfect correlation between station coordinates and pole (X,Y) coordinates. Nevertheless, we still use Σ to compute standard errors in the translation and scale parameters to assess their significance.

In deriving coordinate covariances, geodetic investigators have applied various ad hoc methods, including, for example, constraining a number of coordinates and/or directions to establish a reference frame for the technique in question. We chose to allow our GPS network to remain free-floating until performing the similarity transformation, however the covariance matrix was modified so that covariances are relative to an internally-defined frame, as opposed to the a priori frame.

Correlated errors arising from reference-frame uncertainty are removed from Γ with the application of an orthogonal projection operator $\mathbf{B}$, which projects GPS coordinate variations onto a space orthogonal to a reference frame that is implicitly defined through the initial GPS coordinates:

$$\Gamma_\perp = \mathbf{B}\Gamma\mathbf{B} \qquad (4)$$

where

$$\mathbf{B} = \mathbf{I} - \mathbf{A}\left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T \qquad (5)$$

and where all station coordinates in the initial GPS solution are used to compute matrix elements of $\mathbf{A}$, irrespective of whether they are in the ITRF. This covariance transformation is equivalent to imposing the condition that $\mathbf{A}^T\mathbf{x}$ has zero variance, which intuitively corresponds to fixing the direction, origin and scale of the coordinate axes in an average sense to the initial GPS coordinate solution $\mathbf{x}$ [*Koch*, 1987]. We emphasise that only covariances (not the coordinates) are transformed; we are simply constructing a weight matrix.

$\Gamma_\perp$ is has a 7-rank deficiency because **B** is indempotent. Nevertheless, a weight matrix can be formed from blocks of $\Gamma_\perp$ using the 3×3 covariance submatrices between the coordinates of individual stations. In this study, only the variances are used (for both GPS and ITRF) because only diagonal terms for ITRF were available. In ignoring off-diagonal terms, the relative precision of vertical versus horizontal components is not accounted for in the weight matrix. We still retain the information that some stations are better determined than others. Therefore, the weight matrix is

$$\mathbf{W} = \left[\mathrm{diag}\left(\mathbf{B}\Gamma\mathbf{B} + \Omega\right)\right]^{-1} \qquad (6)$$

where Ω is the ITRF'90 covariance, **B** is defined by equation (5), and Γ is our GPS covariance derived using very weak constraints. Applying equation (3), this weighting method produces estimates and covariances which are insensitive to loose a priori constraints. We scaled the covariance Γ such that the mean formal error in baseline length was $2.8 \times 10^{-9}L$ (where L is length) corresponding to the empirical standard deviation in length differences observed between the first- and second-half of the experiment for lengths > 4000 km.

Tables 2 and 3 list the standard deviations for ITRF'90 and GPS, defined by the diagonal elements of Ω and $\Gamma_\perp$. The larger GPS errors in the southern versus northern hemisphere are due to the lack of stations in the south during GIG'91. The ITRF standard deviations used in this study (as quoted in *IERS* [1991]) do not include the error in propagating the solution to epoch 1991.1, and are therefore slightly underestimated in most cases.

TABLE 3. GPS Solution Transformed into ITRF'90 (Model B) at Epoch 1991.1

SITE NAME	GPS COORDINATE, EPOCH 1991.1 (mm) X	Y	Z	ERROR (mm) X	Y	Z
Sites used in transformation:						
ALGO	918129611	-4346071204	4561977793	8	11	12
FAIR	-2281621290	-1453595749	5756961960	7	6	9
GOLD	-2353614081	-4641385464	3676976522	9	10	9
PASA	-2493304014	-4655215599	3565497323	9	11	9
KAU	-5543838080	-2054587609	2387809514	18	14	13
KOOT	3899225386	396731738	5015078315	12	8	10
MADR	4849202601	-360329229	4114913074	17	11	13
MATE	4641950888	1393056766	4133280284	17	11	13
PINY	-2369510357	-4761207232	3511396075	9	11	9
TROM	2102940474	721569348	5958192071	9	7	15
WETT	4075579393	931807157	4801570924	13	9	11
YARR	-2389025226	5043316891	-3078531021	26	28	20
Other Sites:						
YELL	-1224452355	-2689216066	5633638282	5	6	9
CANB	-4460987984	2682362398	-3674626569	28	27	20
JOHA	5084625596	2670366566	-2768494046	59	40	30
HONE	3132539046	566401749	5508609852	10	7	10
NYAL	1202431415	252626752	6237770882	6	5	12
SIDN	-2327188023	-3522528954	4764832336	8	9	9
SANT	1769724664	-5044512291	-3468396472	41	56	36
SCRI	-2455521635	-4767213472	3441654893	10	12	10
USUD	-3855262601	3427432482	3741020871	23	17	22

Coordinates refers to the GPS monument

Results

Equation (7) shows the estimated translation and scale between GPS and ITRF (Model B), computed using equation (3), with standard deviations for these parameters derived from the initial loosely-contrained GPS covariance for Σ.

$$\begin{aligned} t_x &= (-7.5 \pm 2.6)\,\mathrm{cm} \\ t_y &= (13.0 \pm 2.5)\,\mathrm{cm} \\ t_z &= (-14.8 \pm 13.8)\,\mathrm{cm} \\ s &= (-3.6 \pm 1.3) \times 10^{-9} \\ \theta_x &= (6.0 \pm 530) \times 10^{-9} \\ \theta_y &= (0.4 \pm 530) \times 10^{-9} \\ \theta_z &= (-299.5 \pm 210) \times 10^{-9} \end{aligned} \qquad (7)$$

The frame orientation has no physical significance. The ITRF geocentric origin is generally believed to be accurate at the 2-cm level, therefore the geocentric offset deviates significantly from zero. At this point, we cannot explain this result. The estimated offset closely agrees with *Vigue et al.* [1992], who, using the same data, instead held 3 stations fixed in the SV5 frame and explicitly estimated a geocenter parameter. The scale difference does not appear to be significant since ITRF errors are expected to be at the 10^{-9} level. Results from Model A differ from Model B by <1 cm in origin and $<10^{-10}$ in scale.

The estimated GPS station locations are then transformed into ITRF'90 using the estimated parameters from equation (7). Table 3 lists transformed coordinates for Model B. Table 4 lists the differences between the transformed GPS coordinates and ITRF'90 with combined standard deviations.

The weighted root-mean square (WRMS) difference between GPS and ITRF'90 coordinates is defined as:

$$\mathrm{WRMS} = \left[\frac{\sum_i \left(\widehat{\delta x}_i^{\,2} / \sigma_i^2 \right)}{\sum_i \left(1/\sigma_i^2 \right)} \right]^{\frac{1}{2}} \qquad (8)$$

where $\widehat{\delta x}_i$ is the (GPS-ITRF'90) deviation for coordinate i, and σ_i^2 is the corresponding combined variance. The WRMS between GPS and Model B coordinates is 15 mm. The chi-square of the 7-parameter fit is 33.8 with 29 degrees of

TABLE 4. GPS Coordinate Deviations from ITRF (Model B)

SITE NAME	GPS-ITRF'90 Deviations (mm) X	Y	Z	East	North	Up
ALGO	−19±11	19±13	−1±15	−15	15	−16
FAIR	8±11	2±11	0±13	3	7	−4
GOLD	1±12	−7±14	−1±13	4	−4	4
PASA	21±16	−4±21	−8±17	21	−3	−10
KAUA	−19±20	−24±17	24±16	16	13	13
KOOT	6±21	−16±20	25±20	−16	12	22
MADR	9±23	−6±15	16±21	−5	6	18
MATE	−18±20	15±16	−29±17	20	−14	−28
PINY	5±15	−6±18	0±16	8	−2	3
TROM	6±17	−11±14	27±26	−12	7	26
WETT	−12±16	2±13	−7±14	5	4	−13
YARR	54±28	68±31	−29±24	−78	−6	48

freedom (DOF). Model A gives a WRMS of 17 mm and a chi-square of 44.9. Model B gives much better agreement in the vertical component for the 4 sites which have their vertical velocity set to zero.

Of the 12 stations we have compared, YARRAGADEE has the largest deviation from ITRF (-78 mm in longitude). Without YARRAGADEE included in the fit, station coordinates change at the centimeter-level, and the WRMS is 12 mm, with a chi-square of 21.2 with 26 DOF. In the northern hemisphere, the RMS difference in the North component is 9 mm, in the East is 13 mm, and in the Vertical is 20 mm. (WRMS could not be computed for local coordinates since we used diagonal weights). The removal of YARRAGADEE from the comparison has the biggest effect on the tranformation parameters, but even so the solution is robust: The largest change in the origin offset is 12 mm in the X direction; the scale changes from -3.6×10^{-9} to -4.6×10^{-9}.

Application of Fiducial Constraints

We ask the question, "Do fiducial constraints improve the agreement between the GPS and ITRF solutions?" The answer would be yes if the GPS solution had some inherent internal weakness due to the simultaneous adjustment of satellite orbits and station locations. The answer would be no if the fiducial stations are constrained to coordinates which are in error at the level of the ability of GPS to determine those positions independently.

We repeated the above analysis with the exception that the coordinates of 3 stations, KAUAI, GOLDSTONE, and WETTZELL were constrained to ITRF Model B coordinates. We used the same weight matrix as the free-network solution, solving for the similarity transfomation in the same way. The results are very similar to those shown in Table 4 to within ~3 millimeters, and the chi-square statistic is slightly worse than the free-network case, with a value of 36.9 as compared to 33.8 with 29 DOF. Other selections of fiducial stations produce similar results: Fiducial constraints do not appear to improve the GPS solution.

Conclusions

The RMS difference between transformed GPS and ITRF'90 coordinates is 12 mm in the northern hemisphere, which is as good as any comparison between VLBI and SLR [*Ray et al.*, 1991]. GPS therefore appears to be a viable, independent technique for monitoring geodetic signals from local to global scales. Independent confirmation of geodetic signals would greatly strengthen hypothesis testing in geodynamics investigations, so it behooves us to assess the long-term stability of the GPS reference frame.

For the global GPS data set analyzed here, fiducial constraints do not improve the GPS solution, and are therefore unnecessary. This gives GPS techniques more flexibility and eliminates potential sources of error. For example, site tie errors can lead to network distortion when using fiducial constraints, whereas such errors only affect the free-network solution "externally" without any affect on sensitivity to geophysical signals.

Best results are obtained by mapping the ITRF coordinates to the epoch of our experiment under the hypothesis that the vertical site velocities are zero (accounting for correlations with epoch 1988.0 coordinates using the GLB718 site covariance matrix).

Finally, we conclude that both the ITRF'90 and GPS standard deviations presented in this paper appear to be as good as claimed (at the centimeter level).

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References

Boucher, C. and Z. Altamimi, ITRF 89 and other realizations of the IERS terrestrial reference system for 1989, *IERS Technical Note 6*, Observatoire de Paris, 1991.

Heflin, M.B., W.I. Bertiger, G. Blewitt, A.P. Freedman, K.J. Hurst, S.M. Lichten, U.J. Lindqwister, Y. Vigue, F.H. Webb, T.P. Yunck, and J.F. Zumberge, Global geodesy using GPS without fiducial sites, *Geophys. Res. Lett.*, *19*, 131-134, 1992.

Koch, K.R., Parameter estimation and hypothesis testing in linear models, p. 222, Springer-Verlag, New York, 1987.

IERS, *Annual Report For 1990*, IERS Central Bureau, Observatoire de Paris, 1991.

Larson, K.M., F.H. Webb, and D. Agnew, Application of the Global Positioning System to crustal deformation measurement: 2. The influence of errors in orbit determination networks, *Journ. Geophys. Res.*, 96, (B10), 16567-16584, 1991.

Ray, J.R., C. MA, J.W. Ryan, T.A. Clark, R.J. Eanes, M.M. Watkins, B.E. Schutz, and B.D. Tapley, Comparison of VLBI and SLR geocentric site coordinates, *Geophys. Res. Lett.*, *18*, (2), 231-234, 1991.

Vigue, Y., S.M. Lichten, and G. Blewitt, Precise determination of geocentric station coordinates with the Global Positioning System, in *AGU 1991 Fall Meeting Program and Abstracts*, p. 114, AGU, 1991.

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