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GLOBAL GEODESY USING GPS WITHOUT FIDUCIAL SITES

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Abstract. Baseline lengths and geocentric radii have been determined from GPS data without the use of fiducial sites. Data from the first GPS experiment for the IERS and Geodynamics (GIG '91) have been analyzed with a no-fiducial strategy. A baseline length daily repeatability of 2 mm + 4 parts per billion was obtained for baselines in the northern hemisphere. Comparison of baseline lengths from GPS and the global VLBI solution GLB659 (Caprette et al. 1990) show rms agreement of 2.1 parts per billion. The geocentric radius mean daily repeatability for all sites was 15 cm. Comparison of geocentric radii from GPS and SV5 (Murray et al. 1990) show rms agreement of 3.8 cm. Given n globally distributed stations, the n(n - 1)/2 baseline lengths and n geocentric radii uniquely define a rigid closed polyhedron with a well-defined center of mass. Geodetic information can be obtained by examining the structure of the polyhedron and its change with time.

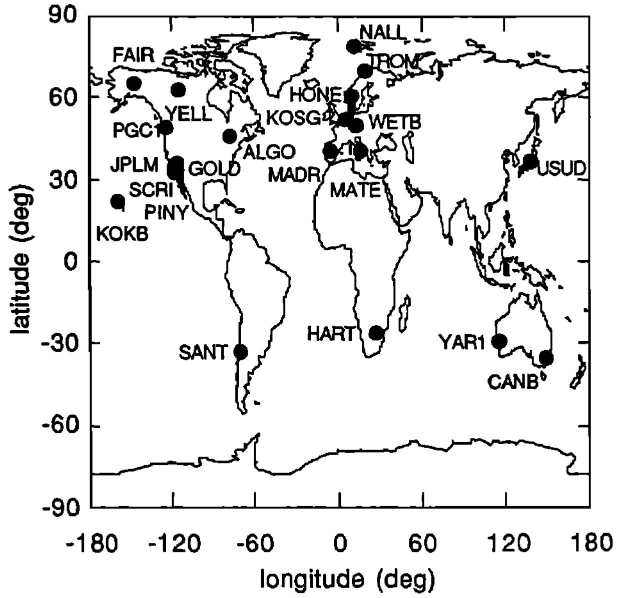

Fig. 1. Rogue receiver locations.

Introduction

The first GPS experiment for the IERS and Geodynamics (GIG '91) took place between January 22 and February 13, 1991. The global extent of the experiment prompted an analysis strategy designed to investigate the amount of geodetic information that GPS can provide without using fiducial sites. We decided to focus on the 21 Rogue receivers which participated. A map with all 21 sites is given in Figure 1. Receiver names and locations are listed in Table 1.

A fiducial site is one with coordinates which are held fixed at previously determined values during analysis. A fiducial network consisting of several fiducial sites can be used to provide a well-defined terrestrial reference frame. Fiducial network strategy has been discussed by Davidson et al. (1985), Malla and Wu (1989), Kornreich Wolf et al. (1990), Larson et. al (1991), and many others. A terrestrial reference frame requires an origin, scale, and orientation. Fiducial networks are not necessary to provide an origin and scale for global GPS measurements. The satellite force model implies an origin at the Earth's center of mass. A scale is implied by the satellite force model, radio propagation model, and GPS data. Furthermore, the orientation provided by fiducial coordinates does not affect rotationally invariant quantities. Baseline length and geocentric radius are rotationally invariant. Given n globally distributed stations, the n(n - 1)/2 baseline lengths and n geocentric radii can be used to construct a rigid closed polyhedron with a center of mass well-defined by the geocentric radii as illustrated in Figure 2. Intrinsic properties of the polyhedron such as the difference between two geocentric radii, the opening angle between two geocentric radii, and the perpendicular distance from the center of mass to a baseline can be examined without the use of fiducial sites.

The absolute orientation of the polyhedron is poorly determined without fiducial sites. The directions of the x-, y-, and z-axes are constrained by placing 10-km a priori standard deviations on each station and initial satellite position component. The direction of the z-axis is further constrained by the signature of Earth's daily rotation. Mathematically speaking, the estimated z-component for each site will have a smaller error than the estimated x- and y-components and all of the errors will be correlated so that rotationally invariant quantities such as geocentric radius and baseline length are well determined. The intrinsic structure of the polyhedron is completely independent of its orientation.

The following analysis demonstrates a no-fiducial approach to global geodesy. Baseline lengths and geocentric radii are determined both precisely and accurately without the use of fiducial sites—the polyhedron is well defined. The polyhedron can be oriented arbitrarily or with information from other techniques such as SLR and VLBI. Consistent information from other techniques such as SLR and VLBI can be used to improve the no-fiducial solution. Our analysis focuses on geodetic parameters. Herring et al. (1991) have obtained polar motion estimates using a no-fiducial approach.

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Table 1 Station Information And Geocentric Radii

Name	Location	Days	Radius (m)	$\hat{\sigma}_R$ (m)
ALGO	Algonquin, Canada	20	6367333.62	0.17
CANB	Canberra, Australia	19	6371684.46	0.12
FAIR	Fairbanks, Alaska	20	6360923.50	0.18
GOLD	Goldstone, California	20	6371978.83	0.13
HART	Hartebeesthoek, S. Africa	16	6375643.95	0.19
HONE	Hønefoss, Norway	16	6362263.30	0.19
JPLM	Pasadena, California	20	6371841.80	0.13
KOKB	Kokee, Hawaii	20	6376292.47	0.08
KOSG	Kootwijk, Netherlands	20	6364932.44	0.17
MADR	Madrid, Spain	20	6370016.64	0.15
MATE	Matera, Italy	20	6369640.56	0.15
NALL	Ny Ålesund, Norway	20	6357629.07	0.20
PGC1	Victoria, Canada	20	6366132.38	0.15
PINY	Pinyon, California	20	6372878.12	0.13
SANT	Santiago, Chile	18	6372503.53	0.12
SCRI	La Jolla, California	16	6371883.45	0.12
TROM	Tromsø, Norway	10	6359486.92	0.15
USUD	Usuda, Japan	14	6372250.91	0.12
WETB	Wettzel, Germany	20	6366607.82	0.16
YAR1	Yarragadee, Australia	20	6373369.65	0.14
YELL	Yellowknife, Canada	20	6361528.66	0.19

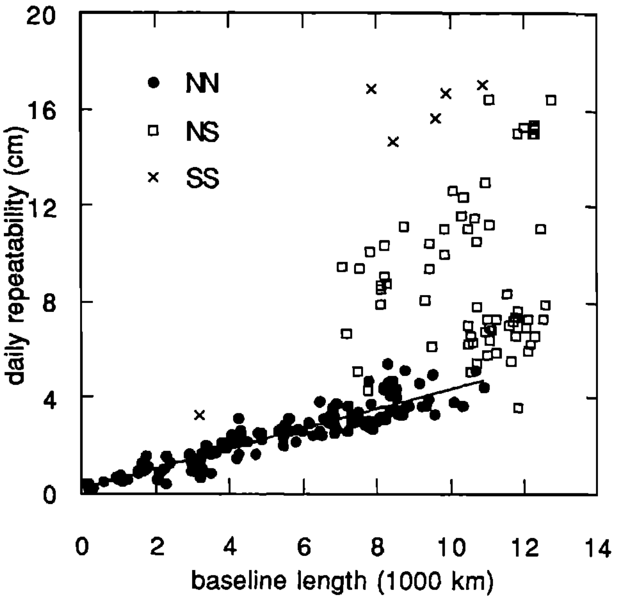

Fig. 3. Daily repeatability versus baseline length. A linear fit to NN baselines yields 2 mm + 4 ppb. A mean value of 9 ppb and 16 ppb was derived for the NS and SS baselines respectively.

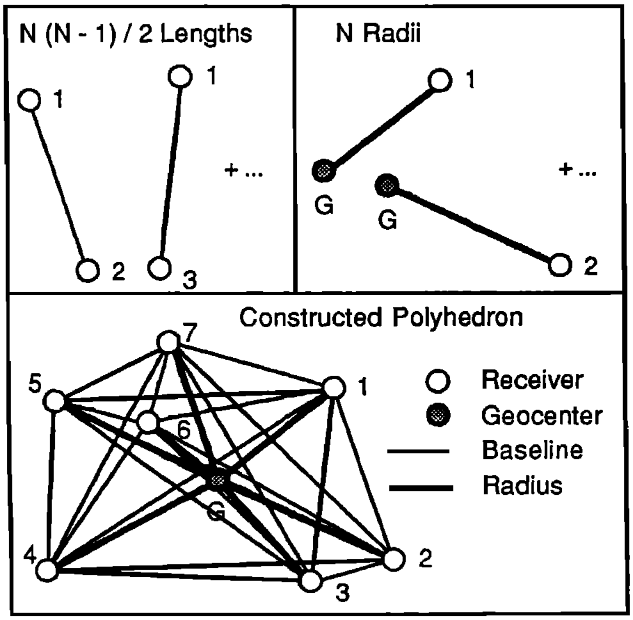

Fig. 2. Given n globally distributed stations, the n(n -1)/2 baseline lengths and n geocentric radii uniquely define a closed polyhedron.

Analysis

Analysis was carried out with the GIPSY (GPS Inferred Positioning SYstem) software developed at the Jet Propulsion Laboratory (Lichten and Border, 1987; Sovers and Border, 1990). That fact to no fiducial sites were used means that no station positions were held fixed. All station positions, initial satellite positions, initial satellite velocities, solar radiation coefficients for each satellite, wet tropospheric zenith delays for each station, carrier phase ambiguities, satellite clocks, and station clocks except the one at GOLD were estimated. All other parameters were held fixed. GIPSY processes undifferenced dual frequency carrier phase and pseudorange data. The assumed noise was 1 cm for carrier phase data and 100 cm for pseudorange data. All station and initial satellite position components had a priori standard deviations of 10 km. Initial satellite velocity components had a priori standard deviations of 10^{-4} km/sec. White noise clock biases were estimated every 6 minutes. Undifferenced carrier phase ambiguities were estimated as real valued model parameters. The elevation angle cutoff was 15 °.

The Earth's dynamic inertial orientation was described by models of precession and nutation along with measurements of UT1 and X and Y polar motion obtained from IERS Bulletins B37 and B38. The Williams (1970) solid Earth tide model was used along with a standard pole tide model and the Pagiatakis (1982) ocean loading model. The Lanyi (1984) tropospheric elevation angle mapping function was used. Wet tropospheric zenith delays were modeled with an a priori standard deviation of 50 cm and a random walk constraint of 1 cm variation per hour. A nominal dry tropospheric zenith delay of 220 cm was used at all sites. Ionospheric delay was removed by taking a linear combination of the dual frequency data measurements. Earth's gravity field was described by the GEM-T2 multipole expansion (Marsh et al. 1990). Only the first 12 moments of GEM-T2 were used. The monopole moment or point mass term was calculated with GM(IERS). The dipole moment was zero because GEM-T2 is geocentric—its origin is at the Earth's center of mass. All higher order moments utilized GM(GEM-T2). The IERS and GEM-T2 values for GM are 398600.440 km^3/sec^2 and 398600.436 km^3/sec^2 respectively. Solar radiation pressure on the satellite was characterized by three solar radiation coefficients. The x- and z-coefficients are proportionality constants which relate the x- and z-solar flux components to the x- and z-satellite acceleration components. The y-coefficient represents a constant acceleration. Here x, y and z refer to a satellite-centered frame with the z-axis pointing radially toward the Earth, the x-axis perpendicular to the z-axis in the Sun-Earth-satellite plane pointing toward the Sun, and the y-axis perpendicular to the x- and z-axes in a right handed sense.

The following simplified description of model prediction illustrates the use of models described above. Satellite positions and velocities are propagated forward in time according to the forces of gravity and solar radiation pressure. Receiver positions are adjusted for tidal effects and transformed into earth-centered inertial coordinates by the precession, nutation, and UTPM values. Satellite to receiver propagation time is calculated from the relative earth-centered inertial positions, clock values, and atmospheric delay in the context of general relativity. Tropospheric delay is calculated from the wet and dry tropospheric zenith delays and the tropospheric elevation angle mapping function. Pseudorange and carrier phase predictions are derived from the propagation time. Estimated parameters are adjusted to make model predictions as consistent as possible with the data in a least squares sense.

Results

Daily output solutions included a vector of all estimated parameters and a full covariance matrix of errors. All days were processed except 1/31/91, 2/2/91, and 2/13/91. The daily position estimate vectors and covariance matrices were combined to yield one final position estimate vector and covariance matrix. The cartesian components of each site position had large errors, roughly 5 m for x and y and 4 cm for z, which were highly correlated. Baseline lengths and geocentric radii were calculated from the final estimate vector and covariance matrix. Figure 3 shows repeatability as a function of baseline length for all 210 baselines. The baselines are divided into three types; those with both sites in the northern hemisphere (NN), those with one site in each hemisphere (NS), and those with both sites in the southern hemisphere (SS). The 17 sites in the northern hemisphere define 136 NN baselines. The 4 sites in the southern hemisphere define 6 SS baselines, leaving 68 NS baselines. A line was fit to the NN baselines, yielding a daily repeatability of 2 mm + 4 parts per billion. A line was fit to the NN baselines because there are enough short NN baselines to determine the offset of 2 mm. The mean value of repeatability over length was 9 parts per billion for NS baselines and 16 parts per billion for SS baselines. The NS and SS baselines are not as well determined as the NN baselines because there are only four sites in the southern hemisphere and common satellite visibility is therefore limited at those sites. Geocentric radii and their daily repeatabilities are listed in Table 1. The geocentric radius mean daily repeatability is 15 cm for all sites.

Figure 4 compares the GPS and VLBI (GLB659) baselines defined by 5 sites with good VLBI ties: JPLM, KOKB, PINY, TROM, and WETB. Error bars represent repeatability divided by the square root of the number of daily measurements. The weighted rms of (GPS - VLBI)/length is 2.1 parts per billion, assuming a conservative combined VLBI and site tie error of 5 mm. The value of 2.5 parts per billion is not changed by assuming a combined VLBI and site tie error greater than 5 mm. Figure 5 compares the GPS and SV5 geocentric radii for six sites with good VLBI or SLR ties: JPLM, KOKB, KOSG, PINY, TROM, and WETB. Error bars again represent repeatability divided by the square root of the number of daily measurements. The weighted rms of (GPS - SV5) radii is 3.8 cm.

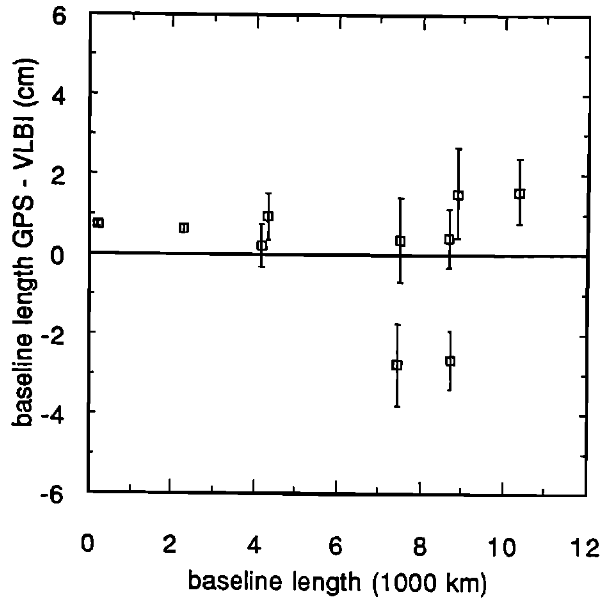

Fig. 4. GPS baseline length solutions compared with VLBI solution GLB659. Stations JPLM, KOKB, PINY, WETB, and TROM were used.

Conclusions

Data from GIG '91 have been analyzed with a no-fiducial strategy. A baseline length daily repeatability of 2 mm + 4 parts per billion was obtained for the NN baselines. Comparison of baseline lengths from GPS and the global VLBI solution GLB659 (Caprette et al. 1990) show agreement of 2.1 parts per billion, assuming a combined VLBI and site tie error or 5 mm. The NS and SS baselines had daily repeatabilities of 9 parts per billion and 16 parts per billion respectively, because there were only four sites in the southern hemisphere and common satellite visibility was therefore limited at those sites. The geocentric radius mean daily repeatability for all sites was 15 cm. Comparison of geocentric radii from GPS and SV5 (Murray et al. 1990) show rms agreement of 3.8 cm. Baseline lengths and geocentric radii derived without the use of fiducial sites were both precise and accurate. Given n globally distributed stations, the n(n - 1)/2 baseline lengths and n geocentric radii uniquely define a rigid closed polyhedron with a center of mass well-defined by the geocentric radii. Geodetic information can be obtained by examining the structure of the polyhedron and its change with time. Intrinsic properties of the polyhedron such as the

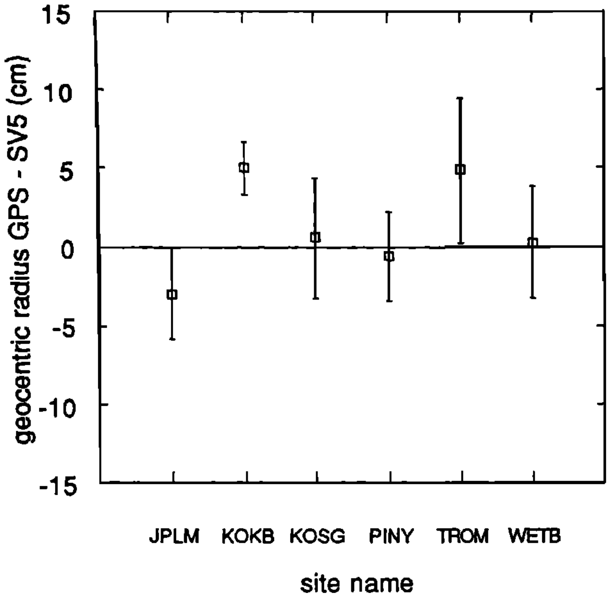

Fig. 5. GPS geocentric radius solutions compared with SV5.

difference between two geocentric radii, the opening angle between two geocentric radii, and the perpendicular distance from a baseline to the center of mass are all well determined. Satellite orbits are also well determined. Baseline repeatability of 2 mm + 4 parts per billion requires very precise orbits. Initial consistency checks have been made by mapping GPS orbits from one day to another and computing rms differences. This test of orbit consistency shows a mean single component rms of 40 cm for February 9 and 10.

The polyhedron has its own scale and origin. Fiducial networks are not necessary to provide an origin and scale for global GPS measurements. Any center of mass and scale information inherent in a given fiducial network may not be consistent with the center of mass and scale implied by the satellite force model, radio propagation model, and GPS data. If the coordinates of two fiducials had been fixed in the above analysis, then the two fiducial geocentric radii and one fiducial baseline would have lead to systematic errors unless they were consistent within 15 cm and 2 mm + 4 parts per billion respectively. The no-fiducial approach also avoids the problem of using several different fiducial networks during an experiment because of missing data at particular fiducial sites. Regional GPS campaigns can utilize the no-fiducial approach by including data from global tracking sites in their analysis.

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