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Research Article

Mapping Dilution of Precision (MDOP) and map-matched GPS

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Abstract. A novel method of map matching using the Global Positioning System (GPS) has been developed which uses digital mapping and height data to augment point position computation. This method reduces the error in position, which is a sum from several sources, including signal delay due to the ionosphere and atmosphere and until recently from 'selective availability' (S/A). S/A was imposed by the US military to degrade purposefully the accuracy of GPS, but was switched off on 2 May 2000, and is to be replaced with 'regional denial capabilities in lieu of global degradation' (Interagency GPS Executive Board, 2000). Taylor *et al.* (2001) describe the Road Reduction Filter (RRF) in detail. RRF is a method of detecting the correct road on which a vehicle is travelling. In the work described here, the position error vector is estimated in a formal least squares procedure, as the vehicle is moving. This estimate is a map-matched correction, that provides an autonomous alternative to DGPS for in-car navigation and fleet management. In this paper, a formula is derived for 'Mapped Dilution of Precision' (MDOP), defined as the theoretical ratio of position precision using map-matched corrections to that using perfect Differential GPS (DGPS) correction. This is shown to be purely a function of route geometry, and is computed for examples of basic road shapes. MDOP is favourable unless the route has less than a few degrees curvature for several kilometres. MDOP can thus provide an objective estimate of positioning precision to a vehicle driver. Precision estimates using MDOP are shown to agree well with 'true' positioning errors determined using high precision (cm) GPS carrier phase techniques. The exact location of a vehicle on a road is essential for accurate surveying applications. These include close range photogrammetry using digital video or still cameras and the verification of digital mapping by measured (GPS and other sensors) trajectories.

1. Introduction

The identification of the particular road on which a vehicle is travelling may be achieved in a number of ways using map matching and other techniques (Scott 1994, Mallet *et al.* 1995, Collier 1990). A particular method developed in earlier work, which is built upon here, solves this identification problem using an algorithmic

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approach. This algorithm is called a Road Reduction Filter (RRF). This RRF computes certain differences (errors) between the trajectory drawn by raw uncorrected GPS receiver positions taken in a moving vehicle and digital road centre-lines. Potential roads are discarded when distance and bearing differences reach certain tolerances. This method will eventually reduce the set of all potential road centre-lines down to just the correct one; within a few seconds in most cases (Taylor and Blewett 2000). What is less certain with this method is the exact location of the vehicle on that road centre-line. That is, the along-track error will vary considerably. The steps of the RRF algorithm are briefly described below:

1. A **Raw** vehicle position is computed using all satellites available plus height aiding, where height is obtained from a DTM, and used to provide an extra equation in the least squares approximation computation, i.e. computation is possible with a minimum of three satellites. For the first epoch all roads (road centre-line segments), which are within 20 m distance (100 m with S/A on) of the computed **Raw** position are selected. It is guaranteed with 95% confidence that the vehicle is on one of these road segments, according to GPS specification (DoD/DoT 1992). The point on each of the *n* road segments that computes the shortest distance to the **Raw** position is selected as the first approximation of the true location of the vehicle, its **Ref** position. That is, there are *n* **Ref** positions used to generate map-matched corrections for use with the next epoch's computed **Raw** position.
2. Map-matched corrections for each satellite pseudorange are computed at each of the *n* **Ref** positions on each road segment for the current epoch, giving *n* different sets of map-matched corrections.
3. The next epoch **Raw** position is computed, as in step 1.
4. Each of the map-matched corrections (step 2) are added to the **Raw** position (step 3) to give *n* **Cor** positions for each *n* road segments.
5. Each of these *n* **Cor** positions is now snapped back onto the nearest road centre-lines to give *n* **Ref** positions. Go to step 2. At each epoch for each of the *n* road segments the distance travelled and bearing between epochs for Raw positions are comparing with these values with equivalent Ref positions. Any road segment where either of these differences is greater than a set tolerance is discarded.

Steps 2 to 5 are repeated continuously. The output point position from the RRF is either taken from the only remaining road centre-line or is the weighted mean of points on all candidate road centre-lines. RRF is fully described with test results in Taylor *et al.* (2001).

2. Least squares estimation of position error vector

One problem with the approach briefly described above is that errors in the GPS signal translate into considerable errors in position. It may be possible at a particular point in time correctly to identify the road a vehicle is travelling on, but the position along the road may be in error by up to 20 m (100 m when S/A was switched on). This 'along-track error' cannot be resolved for a straight road, but it can be resolved if the road changes direction, or if the vehicle turns a corner. A more formal method of computing a map-matched correction is now given, which is then integrated with the Road Reduction Filter (RRF). This map-matched correction, or error vector, is used to adjust the position of the vehicle on the road segment, but only when residual values are low, see equation (9) below. The advantage of formal methods is that quality measures can be derived and used to place confidence bounds for rigorous decision making (for example, to reject road centre-lines that fail a particular hypothesis test). Formal methods also provide insight into the relative importance of factors, which can improve the procedure (e.g. data rates and road geometry).

Figure 1 displays a GPS position at a single epoch. Vector $\underline{b}$ can be considered to be the error vector (position error vector) from the true vehicle position on the road centre-line at grid position **Tru**(E_{Tru}, N_{Tru}) to the uncorrected position computed from GPS at **Raw**(E_{Raw}, N_{Raw}). The perpendicular distance from the **Raw** position to the road centre-line at **Int**(E_{Int}, N_{Int}) is given as L. The road centre-line for this purpose is defined by extending the line segment, which joins previous **Ref** to current **Ref**. The first approximation of the **Tru** position is the **Ref** position, which (as explained in step 5 above) was obtained by snapping the **Cor** position (the **Raw** position corrected using map-matched corrections) onto the closest point on the road centre-line. Furthermore, the observed perpendicular distance from **Raw** position to the road centre-line at **Int** is given by L where:

$$L = \pm\sqrt{(E_{\mathrm{Raw}} - E_{\mathrm{Int}})^2 + (N_{\mathrm{Raw}} - N_{\mathrm{Int}})^2} \tag{1}$$

The positive root of L is taken if the raw point lies to the right of the centre-line, and the negative root if it lies to the left. As L has a sign, it may be better described as a ‘cross track coordinate’ rather than a distance.

Here, L is introduced as a ‘measurement’ which can be modelled geometrically. The model that best fits a series of these measurements provides an estimate of the error vector, $\underline{b}$. Consider the unit vector $\underline{\hat{e}}$ which points normal to the road centre-line (and to the right of the road) at the **Ref** position: the cross track coordinate L

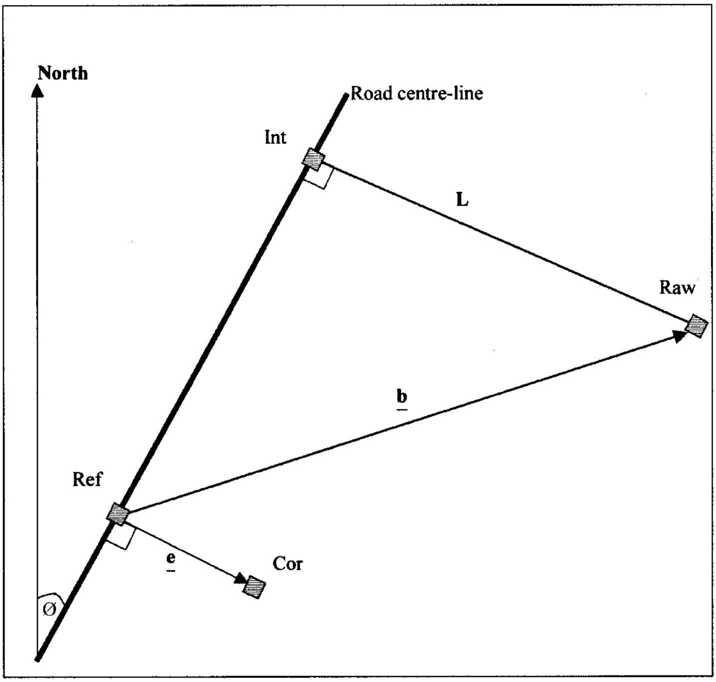


Figure 1. GPS position error vector.

in equation (1) may also be modelled (computed) using the dot product of the two vectors $\underline{b}$ and $\underline{\hat{e}}$

$$\underline{b} = b_E \underline{E} + b_N \underline{N}$$

$$\underline{\hat{e}} = r_N \underline{E} - r_E \underline{N} \tag{2}$$

where $\underline{E}$ and $\underline{N}$ are unit vectors pointing in the East and North directions, and r_E and r_N are the direction cosines of a road segment at the **Ref** position. The **Ref** position is computed using the RRF algorithm. For analytical purposes later (equations (16) (17) and (18)), it is convenient to write them in terms of ϕ, the bearing (clockwise azimuth from North) of the road segment.

$$r_E = \sin\phi$$

$$r_N = \cos\phi \tag{3}$$

Therefore, an observation equation (4) may be formed, where the left side is measured, and the right side is modelled, and includes an unknown term v, which absorbs random position errors:

$$L = b \cdot \underline{\hat{e}} + v$$

$$= b_E r_N - b_N r_E + v \tag{4}$$

Such an equation may be formed each time a GPS raw estimate of position is computed. Now consider n successive GPS raw estimates over a time period where the error vector $\underline{b}$ can be assumed to be approximately constant:

$$L_1 = b_E r_{N1} - b_N r_{E1} + v_1$$

$$L_2 = b_E r_{N2} - b_N r_{E2} + v_2$$

$$L_3 = b_E r_{N3} - b_N r_{E3} + v_3$$

$$\vdots$$

$$L_n = b_E r_{Nn} - b_N r_{En} + v_n \tag{5}$$

In practice, b varies at a level comparable to a road width over 20 to 50 seconds, dependent on the actual road and vehicle velocity. Hence, for GPS raw estimates every second, n can have a value of about 30. This can be written in matrix form:

$$\begin{pmatrix} L_1 \\ L_2 \\ L_3 \\ \cdot \\ \cdot \\ L_n \end{pmatrix} = \begin{pmatrix} r_{N1} & -r_{E1} \\ r_{N2} & -r_{E2} \\ r_{N3} & -r_{E3} \\ \vdots & \vdots \\ r_{Nn} & -r_{En} \end{pmatrix} \begin{pmatrix} b_E \\ b_N \end{pmatrix} + \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ \cdot \\ \cdot \\ v_n \end{pmatrix} \tag{6}$$

This can be written compactly as,

$$L = Ax + v \tag{7}$$

The principles of least squares analysis is applied, a suitable description is given by Blewitt (1997), which minimizes the sum of squares of estimated residuals, giving the following solution for (b_E, b_N):

$$\hat{\mathbf{x}} = (\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}L \tag{8}$$

Note that in equation (8) the co-factor matrix $(\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1}$, also sometimes called the covariance matrix, is implicitly understood to be scaled by the variance of the input observation errors. These errors in this case are characterized by the accuracy of the particular digital road centre-line data used. The focus here is on the co-factor matrix, which like **A**, is purely a function of direction cosines of road segments, i.e. route geometry.

The estimated residuals (misfit of model to the data) are given by:

$$\hat{\mathbf{v}} = L - \mathbf{A}\hat{\mathbf{x}} \tag{9}$$

Equation (9) can be used to assess model fidelity. After the least squares computation it is possible to estimate the precision of the measurements by examining the residuals, i.e. how much the observed values have been altered by the process. If the residual values are low then it is a high precision set of observations (Cross 1994).

Least squares assume that the errors v_i are random with zero mean expected value (i.e. some will be positive, some negative). It does not depend on the errors being normally distributed. This is a reasonable model for GPS pseudorange measurement error, but is not a good model for persistent systematic effects such as atmospheric delay and errors in satellite positions computed from the Navigation Message. However, such systematic effects will be absorbed by the error vector estimate. Note that such persistent effects are not only a common mode for a single receiver's measurements over a short time period, but would also be in common to all GPS stations in the local area. Clearly, the estimated error vector is equivalent to a 'position correction' which could be provided by a local DGPS base station. We call our technique 'map-matched GPS', it does not require data from another GPS base station, but provides the same type of position correction.

Note that the GPS data and the digital map data have been incorporated into this formal scheme through the 'measurement' of L. An advantage of taking such a formal approach to map matching can therefore be seen as the quantification of expected errors, which can in turn be used to narrow down the search for possible positions. For example, alternative hypotheses where a vehicle may have taken one of three roads at a junction can be assessed in terms of the level of estimated residuals, as compared to the level of expected errors.

The modelled error in the determination of the error vector can be found from the covariance matrix, which can then be used to plot a confidence ellipse within which the true value of error bias can be expected to lie. The covariance matrix is computed as:

$$\mathrm{C} = \sigma^2(\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1} \tag{10}$$

The constant σ^2 represents the variance in raw GPS positions, excluding the effects of common mode errors. In other words, σ should equal the standard deviation in raw GPS positions if perfect DGPS corrections were used to remove the effects of non-random common model errors. Its value tends to be dominated by signal multipath around the vehicle, and varies with the geometry of the satellite positions, an effect known as 'horizontal dilution of precision' (HDOP). Typical values are at the meter level. One possibility is to use the estimated residuals themselves to estimate the level of σ. However, this would be inadvisable because it is intended to use C to test the significance of high levels of residuals, which would have created a circular argument.

3. Quantifying road geometry: Mapping Dilution of Precision (MDOP)

Equation (10) given above for the computation of the covariance matrix leads to an elegant method of quantifying road geometry as to its suitability for estimating error in position on-the-fly.

First note that the least squares method assumes that the 'co-factor matrix' $(\mathbf{A}^T\mathbf{A})^{-1}$ exists. It is a necessary but not sufficient requirement that $n \geqslant 2$. If the two **Ref** positions are collinear (the road is perfectly straight), then a third position is required that is not collinear. In the work here $n = 30$. We now explore how the co-factor matrix can be interpreted, and how it is related to the shape of the road.

The diagonal elements of the cofactor matrix can each be interpreted from equation (10) as the ratio of the error squared in estimated error vector component to the expected error squared of a single GPS position in the case that an ideal DGPS position correction were used. To obtain a single number that relates to standard deviation of position instead of variances and covariances, we follow the example of classic GPS theory by which the square root of the trace of the cofactor matrix is taken as a 'Dilution of Precision' (DOP) value. We therefore define 'Correction Dilution of Precision' (CDOP) as:

$$\mathrm{CDOP} = \sqrt{\mathbf{Tr}(\mathbf{A}^T\mathbf{A})^{-1}} \tag{11}$$

From the definition of matrix **A** in equation (6), we can write CDOP in terms of the direction cosines at each of the sampled points on the road. Starting with the cofactor matrix:

$$(\mathbf{A}^T\mathbf{A})^{-1} = \begin{pmatrix} \sum\limits_n r_{Ni}^2 & -\sum\limits_n r_{Ei} r_{Ni} \\ -\sum\limits_n r_{Ei} r_{Ni} & \sum\limits_n r_{Ei}^2 \end{pmatrix}^{-1} = \frac{\begin{pmatrix} \sum\limits_n r_{Ei}^2 & \sum\limits_n r_{Ei} r_{Ni} \\ \sum\limits_n r_{Ei} r_{Ni} & \sum\limits_n r_{Ni}^2 \end{pmatrix}}{\sum\limits_n r_{Ei}^2 \sum\limits_n r_{Ni}^2 - \left(\sum\limits_n r_{Ei} r_{Ni} \right)^2} \tag{12}$$

Therefore equations (12) into (11) gives

$$\mathrm{CDOP} = \left(\frac{\sum\limits_n r_{Ni}^2 + \sum\limits_n r_{Ei}^2}{\sum\limits_n r_{Ei}^2 \sum\limits_n r_{Ni}^2 - \left(\sum\limits_n r_{Ei} r_{Ni} \right)^2} \right)^{1/2} \tag{13}$$

From equation (3), the numerator is simply n, so the whole formula can be reduced to:

$$\mathrm{CDOP} = n^{-1/2} \left(\overline{r_{Ei}^2}\, \overline{r_{Ni}^2} - \overline{r_{Ei} r_{Ni}}^{\,2} \right)^{-1/2} \tag{14}$$

where the overbars denote averaging over the section of road (for which the error is assumed to be approximately constant). CDOP therefore depends on road geometry, and will be inversely proportional to the number of GPS measurements n taken over a fixed time interval. With enough measurements and with sufficient change in road direction, it is possible to reduce CDOP to <1.

Note that GPS data recording should be sufficient to sample any detail in road shape that is present in the digital map. It is therefore preferable to record GPS data at a high rate, e.g. 1 per second. Going at higher rates than this will not help particularly because of time-correlated errors in multipathing, and because at this rate the road is approximately straight between points. Where there is detailed road shape the rate of sampling will increase naturally owing to necessary reductions in vehicle velocity.

A related quality measure is 'Mapping Dilution of Precision', (MDOP) which we define as the ratio of position precision using map-matched GPS to that using perfect DGPS corrections. In this case, we assume that if n is much greater than 1, then the map-matched correction (i.e. the error vector) is uncorrelated with the error in any single data point. Therefore, the corrected position will have a variance equal to the variance in the perfect case plus the variance in the correction. As this is to be divided by the variance in the perfect case, the result is:

$$\begin{aligned} \mathrm{MDOP} &= 1 + \mathrm{CDOP} \\ &= 1 + \sqrt{\mathbf{Tr}(\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1}} \end{aligned} \tag{15}$$

This measure is particularly useful because:

- it is easily interpreted as a 'level of degradation' in precision as a result of not using a perfect DGPS base station
- it can be tested for validity under controlled conditions.

As we shall describe, testing was carried out using an ultra precise GPS method (e.g. carrier phase positioning) to determine the true level of corrected position errors, and then compare this with the errors obtained by applying a near-perfect DGPS correction. The point is that equation (15) can be computed easily in real time (even ahead of time!) by simply knowing the road shape.

Note that MDOP is always greater than 1 because comparison is made with perfect DGPS. It is worth keeping in mind that no DGPS system is perfect; hence $\mathrm{MDOP} > 1$ does not necessarily mean that real DGPS will give better results than map-matched GPS.

4. MDOP for basic road shapes

From equations (3), (14) and (15), we can write MDOP analytically in terms of the direction cosines of the vector normal to the road.

$$\mathrm{MDOP} = 1 + n^{-1/2}\left(\overline{\sin^2\phi}\;\overline{\cos^2\phi} - \overline{\sin\phi\cos\phi}^2\right)^{-1/2} \tag{16}$$

This equation can be rearranged into the following form:

$$\mathrm{MDOP} = 1 + 2n^{-1/2}\left(1 - \overline{\cos 2\phi}^2 - \overline{\sin 2\phi}^2\right)^{-1/2} \tag{17}$$

The first thing to note about MDOP is that it takes on the following maximum (worst case) and minimum (optimum) values:

$$\begin{aligned} &\mathrm{MDOP}_{\max} = \infty\,; \quad \phi = \text{constant} \\ &\mathrm{MDOP}_{\min} = 1 + 2n^{-1/2}; \quad \overline{\cos 2\phi} = \overline{\sin 2\phi} = 0 \end{aligned} \tag{18}$$

The maximum condition is satisfied for a straight road. As we shall see, the minimum condition is satisfied for the simple case of a right-angled bend. Keeping in mind the definition of MDOP, we see that GPS error ceases to be a dominant error source when MDOP$\leqslant$2, which the above equation satisfies when using four GPS measurements around a right-angled bend. As more measurements are introduced, MDOP approaches 1, which implies that positioning is as good as using a perfect DGPS system.

Equation (17) can be easily computed for any road using a graphical interpretation of the term we call the 'path closure ratio':

$$S(\vartheta)=\overline{\cos\vartheta}^2+\overline{\sin\vartheta}^2 \tag{19}$$

Consider a path constructed using segments i each of equal length and with bearing ϑ_i (figure 2). The path closure ratio S can be shown to be equal to the square of the ratio of straight-line distance between the starting and end points D to the total path length P:

$$S(\vartheta_i)=(D/P)^2 \tag{20}$$

Obviously, S ranges from 0 to 1. We can therefore take our digital map of the road, and transform it to a path where all of the path segments have double the bearing of the real road, and where each road segment between GPS points are mapped into segments of equal length. We can then compute MDOP as follows:

$$\begin{aligned}\text{MDOP}&=1+2n^{-1/2}(1-S(2\phi))^{-1/2}\\&=1+2/\sqrt{n(1-(D/P)^2)}\end{aligned} \tag{21}$$

Note that a path of fixed length P is therefore equivalent to a road section covered in a fixed amount of time (because GPS data are recorded at equal intervals). So for a fixed amount of time, the path which ends closest to the starting point produces a smaller value of S, and a smaller (more favourable) value of MDOP.

This graphical method is so powerful, that results can be visualized without any computation (figure 3). For example, a sharp right-angled bend in a road will map onto a path which doubles back on itself, reducing S to zero, and hence producing the minimum value of MDOP. A road that gently sweeps though 90° will map onto a path that heads back in the opposite direction, but is displaced by some distance, and therefore will produce good, but not optimum results. A road that moves in a semi-circle (e.g. around a large roundabout) will map into a path that is a complete circle, and hence will produce optimum results.

Table 1 summarizes the results for the computation of the path closure ratio

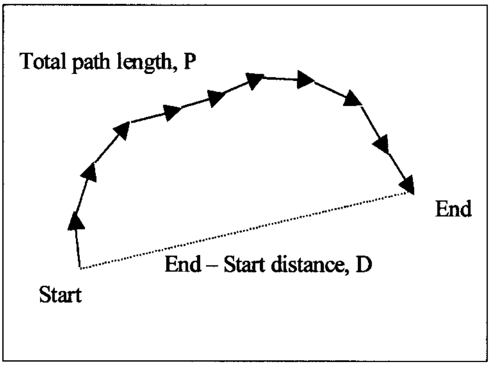


Figure 2. Path constructed of unit vectors.

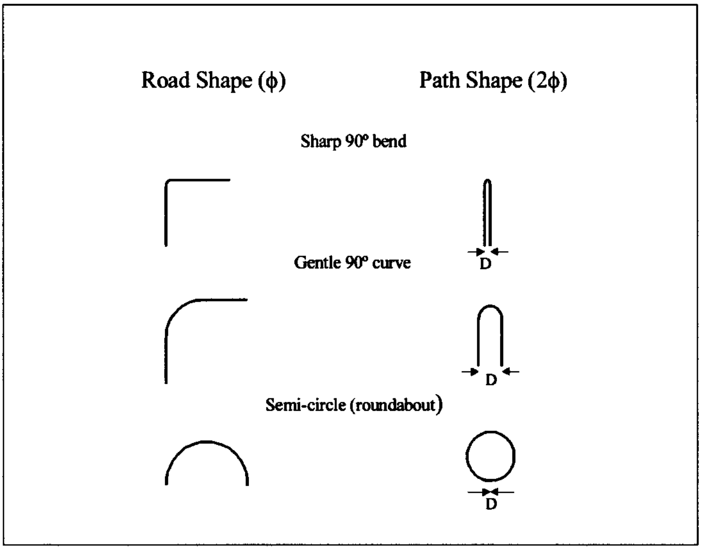


Figure 3. Basic road shapes can be transformed into path shapes with twice the curvature, which can then be interpreted in terms of favourable geometry (MDOP).

Table 1. Quality measures associated with various road geometries for map matched GPS.

Road shape description	Path closure ratio $S(2\phi)$	Mapping dilution of precision, MDOP	Resolution time T (sec)
Instant bend, 10°	0.97	$1+11.5/\sqrt{n}$	133
Instant bend, 20°	0.88	$1+5.8/\sqrt{n}$	34
Instant bend, 45°	0.5	$1+2.8/\sqrt{n}$	8
Instant bend, 90°	0	$1+2/\sqrt{n}$	4
Instant bend, angle α	$\cos^2\alpha$	$1+2/\sin\alpha\sqrt{n}$	$4/\sin^2\alpha$
Smoothest curve, 10°	0.99	$1+19.9/\sqrt{n}$	396
Smoothest curve, 20°	0.96	$1+10.0/\sqrt{n}$	100
Smoothest curve, 45°	$8/\pi^2=0.81$	$1+4.6/\sqrt{n}$	22
Smoothest curve, 90°	$4/\pi^2=0.41$	$1+2.6/\sqrt{n}$	7
Smoothest curve, α	$\sin\alpha/\alpha^2$	$1+2/\sqrt{(1-\sin^2\alpha/\alpha^2)n}$	$4/(1-\sin^2\alpha/\alpha)$

$S(2\phi)$ for various road shapes, which can then be inserted into equation (21) to find the appropriate MDOP value. Also given is the value of n, which would be required to bring the MDOP value <2. We call this number the 'resolution time' T, since it tells us how many data intervals are required to bring GPS error to a level below that expected from random position errors. Under the assumption that we use 1 second GPS data, T is in seconds. Alternatively, the value in the final column of table 1 may be considered to be the minimum number of data points required to describe each road shape, in order to evaluate MDOP.

5. Testing MDOP

To evaluate the effectiveness of MDOP, GPS C/A code observation data were collected in a vehicle driven on roads in the suburbs of Newcastle-upon-Tyne, UK (see figure 4). Over the same period, dual frequency phase data were collected in the vehicle and also by a static receiver recording base station data on the roof of the Department of Geomatics, University of Newcastle. These dual frequency data were used to compute a high precision (cm accuracy) GPS solution, which was assumed to be the 'true' position of the vehicle at each epoch (second). The details of all hardware, software, data sets and processing techniques are given in Taylor *et al.* (2000). All available satellites visible to both receivers were used in the position solution computation (no elevation mask), this number varied throughout the route from none to eight. Three point position solutions were computed:

1. RAW solution—using C/A code data.
2. Map-matched GPS solution—using C/A code data, the RRF, MDOP and digital map data.
3. RTK solution—using dual frequency phase data from both the vehicle and the base station to compute a high precision (cm accuracy) GPS solution. The 'true' position of the vehicle at each epoch was assumed to be that given by this solution.

The map-matched GPS positions output from method 2 used Ordnance Survey (OS) road centre-line data, OS DTM data for height aiding, RRF for correct road selection and MDOP to correct for along track errors. To display the vehicle positions correctly on OS large scale mapping all resultant latitude, longitude and height coordinates from the three solutions were transformed to OSGB36 (Ordnance Survey of Great Britain 1936) National Grid, with a nominal transformation accuracy of 20 cm (OS 1999). At each epoch, where all three positions were available, the difference in position between RAW and RTK and map-matched GPS and RTK were calculated (Position Error).

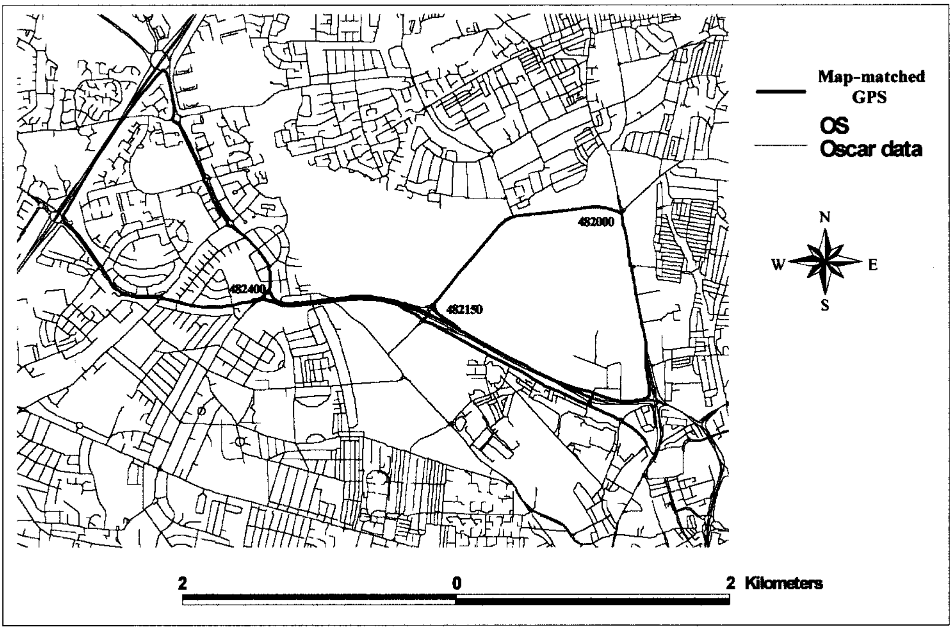


Figure 4. Test route, displaying GPS seconds.

A summary of the results is given in table 2. It is also of interest to note that the maximum position errors were 177 m for the RAW data and 76 m for the map-matched GPS data (43 m for 95%). Mean error of position has been reduced from 36 m to 13 m over a total of 1112 vehicle positions. The variation in both cross track and along track error is also much reduced. It can be seen that the map-matched GPS described in this paper provides a much-improved accuracy of position, particularly if the worst 5% of position errors are removed. In fact, Mapping Dilution of Precision can be used to identify (predict) where on the route the error vector will be least accurately modelled. Inspection of the estimated residuals, equation (9), tells us when we have a poor error vector. If the residuals are low then we can reject a road segment with the RRF.

All the really large errors occur when the vehicle is stationary or almost stationary such as at a road junction, e.g. approximately at GPS seconds 482 000, 482 400 (both at roundabouts) and at 482 150 (motorway slip road). These positions can be seen on the map in figure 4 and the corresponding errors in figure 5. The only other times are at the beginning and end of the route for the same reason. If we ignore these times when the positions are in gross error, it can be seen in figure 6 that cross-track errors are almost always small, because the car has been positioned by map-matched GPS on the correct road. Along-track errors are larger, as expected, because once a correct road is identified, it takes a number of epochs before the algorithm can successfully use MDOP to correct the position; see table 1.

A second set of data was collected over the same route, with exactly the same equipment and operational parameters. These data were processed in the same manner as described above. The only significant difference being that the data were collected after S/A had been switched off. A summary of the results, are given in table 3. Again, it is interesting to note the maximum position errors were 36 m for the RAW data and 21 m for the map-matched GPS data (7 m for 95%).

Table 2. Statistics summary of data collected when S/A was switched on.

	Map-matched DGPS				RAW
	Cross-track error (m)	Along-track error (m)	Position error (m)	Position error [95%] (m)	Position error (m)
Mean	−1.0	−2.1	12.7	9.3	35.727
Standard Deviation	8.733	17.548	18.9	11.3	42.842

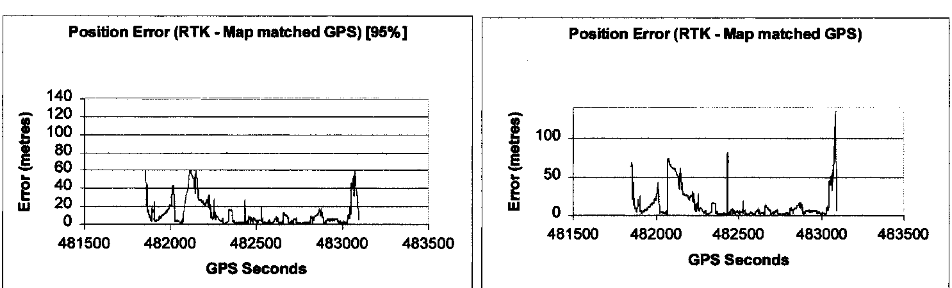
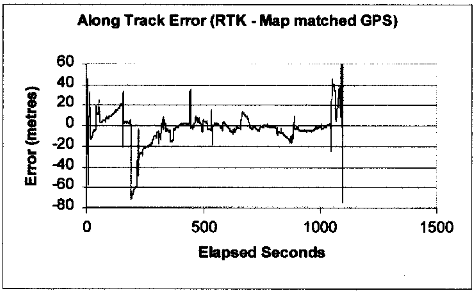


Figure 5. Position errors for map-matched GPS.



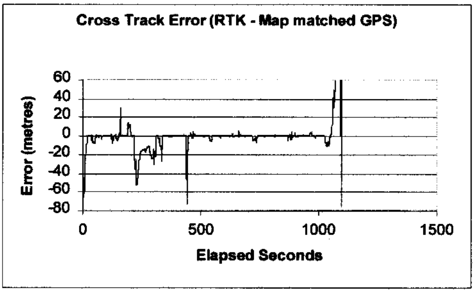


Figure 6. Cross-track and along-track errors for map-matched GPS.

Table 3. Statistics summary of data collected when S/A was switched off.

	Map-matched DGPS				RAW
	Cross-track error (m)	Along-track error (m)	Position error (m)	Position error [95%] (m)	Position error (m)
Mean	−0.28	0.22	2.40	2.02	4.58
Standard Deviation	1.25	2.27	2.44	1.65	2.46

6. Conclusions

From table 1 we can see that the position error can be resolved to within the expected random error of perfect DGPS for all except the slightest of change in road geometry. Problems begin to arise with roads, which curve by only 20° within the period that the error is assumed to be constant (~30 sec for road navigation), although even 10° are sufficient provided the bend is effectively instantaneous. We therefore conclude that only if roads are straighter than 10–20° during a 30 second driving period (i.e. 0.4–1 km in typical driving conditions) will map-matched GPS be significantly worse than DGPS. However, the full precision of DGPS is certainly not required for finding the correct road centre-line, so these numbers are in any case extremely conservative for that purpose. In summary, we expect on firm theoretical grounds that combined RRF and map-matched GPS to be as good as DGPS for correct road centre-line identification in almost any possible circumstance. This has the distinct advantage of being a completely self-contained system, requiring no radio communication for differential corrections and continuous data provision. Furthermore, because the computation of the estimated GPS receiver position is part of the RRF and a digital terrain model derived height aiding is used in the solution, only three satellites are necessary for a solution.

Envisaged further work will include extensive field-testing of the combined map matched GPS and RRF approach to vehicle tracking. The development of the RRF algorithm, to include network connectivity checking and to use map intelligence, such as drive restriction information. Moreover, an investigation of other techniques to reduce the number of satellites required for a solution will be made. Bullock *et al.* (1996), examined two satellite tracking for urban canyons and map matching where only a two-dimensional position is required.

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